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N.E. NEBA-FABS

A STUDY ON THE PERFORMANCE OF PASSIVELY HEATED SOLAR HOUSES

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ABSTRACT

In this paper , analytical techniques are developed for evaluating the performance of passively heated buildings. As most buildings use conservatories to enhance their performance , the direct gain types having attached conservatories are considered in detail. The complexity of the problem is minimized by ensuring all equations are developed from first principles.

Auxilliary energy predictions for space heating by the method is in close agreement with monitored data for three occupied houses in Milton Keynes in England.

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NOMENCLATURE

A	Free area of inlet opening in square metres eqn. (2.46).
B	Proportion of heat transfer air in conservatory relative to the direct gain system. Air proportion in conservatory equals $(1 - B)$ eqn. (1.7.1).
F	$(1 - F)$ is the fraction of transmitted insolation Q_T absorbed by storage and does not heat room air. eqn.(1.3.9).
g	The gravitational constant $g = 9.81\ m/s^2$.
K	A constant term defined in (A13).
K^*	The inverse of K. (A13).
κ	The thermal conductivity of the MSW (massive storage wall) material eqn. (1.6.3). $Wm^{-1}K^{-1}$
$\bar{L}$	The average daily total heating season load. (eqn. 1.2.2). Watts
L	The wall thickness of the MSW house eqn. (1.6.3). [m]
N	Number of days in the heating season. eqn. (1.2.2).
SHF	The fraction of house heating load which is supplied by solar energy.
$\overline{SLR}$	The ratio of the insolation transmitted into the house, Q_T, divided by the house heating load $\bar{L}$.
x	The solar load ratio (SLR).

Subscripts

A_i The inlet area (i.e. area of window or door in conservatory) in m^2 eqn.(2.51).

A_{out} The outlet area (i.e. a stack area in roof) in m^2 eqn. (2.51).

C_o A functional coefficient dependent on climatic as well as building paramters eqn. (1.2.1).

C_o^n The functional coefficient C_o during nightime periods as defined in eqn. (1.3.5).

C^* A wind pressure coefficient appearing in eqn. (2.34).

C_p The specific heat capacity of air at constant pressure, taken as C_p = 1012J/Kg K. eqn. (2.67).

$E_d^{(1)}$ $E_n^{(1)}$ The energy lost from storage for a water wall house to ambient for a given twenty-four hour day during daytime and night time periods respectively, defined by eqns. (1.5.10) and (1.5.11).

F_c Fraction of transmitted insolation Q_T, through conservatory not absorbed by storage and hence directly heats air in conservatory. Fraction directly heating air in conservatory F_c > fraction directly heating air in house, due to storage mass in house. (section 1.9).

F_m The actual flow multiple on flow due to temperature difference for combined vent flow by wind and thermal forces (A.17).

F_R The equivalent heat removal factor for the building envelop (dimension less ratio) eqn. (2.68).

$\bar{L}_d$ The average daytime heating season load eqn. (1.2.2). Watts

$\bar{L}/\bar{L}_d$	Ratio average daily load to daytime load.
$\bar{L}_n$	The average night time heating season load (eqn. 1.2.2). Watts
L_t	The total heating load over time t. eqn. (1.1.1). Watts
L_1, L_2, L_3	The terms L_1, L_2, L_3, represent the contribution of the water wall to the heating load for a given day. eqns. (1.5.14), (1.5.13) and (1.5.15) respectively. Watts
$\dot{M}_I$	Infiltration air leakage rate into building in kg/sm² of crack area A_o. eqn. (2.34).
(mc)s	The heat content of the storage as in eqn. (1.3.9). [J/m²K]
$\dot{M}_{vent}$	Rate of ventilation air flow into building by wind or thermal forces. section (2.2.2). kg/s
$\dot{m}_{vent,st}$	Mass flow rate of air due to thermal forces in (Kg/s) (by ventilation) eqn. (2.55).
$\dot{m}_{vent,w}$	Ventilation mass flow rate of air into building due to wind forces in Kg/s eqn. (2.54).
P_{atm}	The atmospheric pressure in N/m², taken as P_{atm} = 1.01325E05 (N/m²).
P_i	The static pressure at point i in Fig.2.6 (N/m²) as defined by eqn. (2.25).
P_o	The static pressure at point o in Fig. 2.6 (N/m²) as defined by eqn. (2.23).

$Qd^{(1)}, Qn^{(1)}$	The energy collected by room air during daytime and nightime on a given 24 hour day defined by eqns (1.3.15) and (1.3.16) respectively.
Qd^{1*}, Qn^{1*}	The energy collected by room air at day-time and night time periods respectively for a direct gain system with attached conservatory and no night time isulation. eqn. (1.7.20). § appendix 11.
Q_R	Q_R in storage is that fraction of Q_T which is not put to use on that same day, in heating room air; at nightime or daytime. Eqn (1.3.19).
Q_T	Vertical insolation transmitted through vertical double glazing in J/m^2 . section 1.9.
$\dot{Q}_u$	The useful energy supplied by the passive solar element eqn. (1.2.2): [Watts] or [Joules].
$q_u(t')$	The useful solar gain per unit time eqn. (1.1.1). [J/s] or [Watts].
R_a	The universal gas constant for air taken as R_a = 287.045 J/Kg K air. eqn. (2.55).
R_{oi}	The crack resistance or resistance offered by the crack in m^3 /s of crack area for the case considered (i.e. all types of windows and sliding glass doors) eqn. (2.37).
r_c	Constant; A correlation coefficient (A.16).
S_A	The mass surface absorbance of storage (section 1.9).

$SHF_d^{(o)}$	SHF values for daytime periods during which backup is required eqn. (1.2.3)
SHF^{day}	SHF during daytime periods as defined by eqn. (1.3.7).
SHF^{night}	SHF during nightime periods as defined by eqn. (1.3.8).
$SHF_n^{(o)}$	SHF values for nightime periods during which backup is required. eqn. (1.2.4).
$SLR_{min,d}$	The points at which $SHF_d^{(o)}$ becomes unity eqn.(1.2.6).
T_a	Ambient temperature. [°K]
$T_{a,c}$	A temperature variable defined by eqns. (2.55d) in ° K.
T_c	Thermostatically set temperature of room air at and not below T_c = 18.3°C.
$\bar{T}_{co}^d$	Average daytime temperature, in conservatory during heating season (section 1.9) [°K]
$\bar{T}_{co}^n$	Night time average temperature in conservatory during heating season (section 1.9)[°K] .
T_s^d, T_s^n	The storage temperature, superscript d & n denotes daytime and nightime as in eqn. (1.3.9). [°K]
$T_s(o)$	The storage temperature at the beginning of the day eqn. (1.3.10). [°K]
$T_s^d(\Delta t^d)$	The value of storage temperature T_s at the beginning of nightime eqn.(1.3.13).[°K]

t' Any given time interval eqn.(1.1.1).

t^d, t^n Denotes daytime and night-time respectively in eqn.(1.3.9).

Δt^d, Δt^n Denotes length of daytime and night-time respectively in eqn. (1.3.9).

$U_{co,s}$ Heat tranfer coefficient from conservatory to storage wall (section 1.9) in $Wm^{-2}K^{-1}$.

U_{eq} The equivalent heat transfer coefficient for the building envelop defined by eqn. (2.69). $Wm^{-2}K$

U_{SA} The heat transfer coefficient from storage to room air. eqn. (1.3.9). $Wm^{-2}K$

U_{sa}^d U_{sa}^n The daytime and night time heat transfer coefficients from storage to ambient for a water wall house, Eqns. (1.5.2) and (1.5.5) respectively. $Wm^{-2}K$

U_{sw} Heat transfer coefficient from storage to room air, through the internal storage surface for a MSW house. eqn. (1.6.2). $Wm^{-1}K^{-1}$

U_{vent} An effective heat transfer cxoefficient which approximately represents the effect of venting in a MSW house. eqn. (1.6.2). $Wm^{-2}K^{-1}$

U_w^d The daytime heat transfer coefficient from storage to room air for a MSW house given by eqn. (1.6.2). $Wm^{-2}K^{-1}$

$\overline{V}_{season}$ The average of seasonal wind velocity as defined in eqn. (2.45). m/s

V_{wind}(Vw) Average wind velocity as defined in eqn. (2.45). m/s

Greek Symbols.

α	The fraction of transmitted insolation, Q_T which is absorbed by storage for a water wall house. eqn. (1.5.1).
δ	The point to or from the mean SLR (x) at which the distribution function $\rho(x)$ vanishes eqn(1.3.1).
$\rho(x)$	A function which describes the frequency of occurrence of different values of the ratio of daily insolation to load (x), over the course of a heating season eqn. (1.3.1).
ρ_i	The density of indoor air (Kg/m^3) eqn. (2.25).
ρ_o	The density of outdoor air (kg/m^3) eqn. (2.23).
η	A parameter in eqn. (1.7.18) defined by eqn. (1.7.20).
τ	The relaxation time for heat transfer from storage to room air defined by eqn. (1.3.11), (hrs).
τ^d , τ^n	The relaxation time τ during daytime and nightime periods (hrs).
ω	Hour angle.
ω_s	Sunrise (sunset) hour angle.

SECTION 1 - 2

FORMALIZATION OF THE DIRECT GAIN HOUSE AND DIRECT GAIN HOUSE WITH SOLARIUM OR CONSERVATORY

1. Theoretical Consideration of the Passively-Heated Solar House.

1.1 Fundamentals of the Problem.

The object of the computations is to express a solar heating fraction, SHF, of a house in terms of the Solar Load ratio, as well as building and climatic parameters.

(a) Definitions

(i) Solar Heating Fraction SHF

SHF is defined as the fraction of a house heating load which is supplied by Solar Energy. Over a given period of time, t, (which can be the entire heating season or a sizeable fraction thereof, e.g. a month) SHF is given by

$$SHF = Q_u/L_t = (1/L_t)\int_0^t q_u(t')\,dt' \qquad (1.1.1)$$

where L_t is the total heating load over time, t, Qu is the useful solar gain collected over time, t, and $q_u(t)$ is the useful solar gain per unit time. In order to be specific the period of time to be considered henceforth will be the entire heating season. The useful solar gain is the solar energy gain which does not result in heating room air above a specific comfort temperature, T_c. The house is assumed to have a backup heating system which ensures that room temperature never falls below T_c. It is also assumed ventilation is employed to maintain room temperature at T_c, whensoever the room air temperature exceeds T_c. If the heating load is calculated as specified above, then the effect on SHF of activating ventilative cooling of a temperature slightly above T_c is negligible. (8)

(ii) Solar Load Ratio SLR

SLR is defined as the ratio of insolation transmitted into the house, Q_T, divided by the house heating load, L. The insolation transmitted through the vertical south-facing glazing can be readily calculated from the more commonly available data of insolation on a horizontal surface (9).

1.2 Energy Book Keeping

In a case where there was no overheating at any time during the heating season, then all collected energy would be useful energy, and SHF would be proportional to SLR.

$$\mathrm{SHF} = \mathrm{Co}\ \mathrm{SLR} \qquad - 1.2.1.$$

The function Co is a function of climatic as well as building parameters. The equation 1.2.1. is general throughout the value of Co and its dependence on building parameters vary with the type of passive heating element. For sufficiently large SLR values, SHF becomes unity. Due to overheating during part of the heating season, not all collected energy is useful energy (energy is dumped) and SHF averaged over the heating season is less than the value predicted by the linear form of equation 1.2.1. Hence, there is a non-linear dependence of SHF on SLR in a "transition" region characterised by intermediate SLR values. Solar heating fraction is therefore linear in SLR at low SLR values, non-linear in a "transition" region of intermediate SLR values and unity at large SLR values. An accurate "energy book keeping" for determining what fraction of the time overheating occurs and what magnitude of collected energy is dumped is therefore essential for calculating SHF in the transition region.

For purposes of energy book keeping, the heating season is divided into periods during which solar gains provides all of the heating load (no backup required) and periods during which solar gain provides only part of the heating load. A relatively coarse division of time into days and nights (rather than the hourly calculations usually used in numerical simulation), is employed for energy book keeping. The computations are then further divided into days (or nights) during which solar gain (or the storage thereof) provides all of the load, and days (or nights) during which solar gain provides only part of the load. It is assumed that the average daily load $\bar{L}$ that is represented by the daytime load Ld, and the night time load, Ln, does not vary appreciably over the heating season. SHF can then be expressed as a weighted average over daytime and night time periods during which back-up heating is either required or not required.

$$SHF = (1/N)(\bar{L}_d/\bar{L}) \sum_{\substack{\text{days of no}\\ \text{backup}}} 1 + (1/N)(\bar{L}_d/\bar{L}) \sum_{\substack{\text{days with}\\ \text{backup}}} (Q_u/L_d)$$

$$+ (1/N)(\bar{L}_n/\bar{L}) \sum_{\substack{\text{nights of no}\\ \text{backup}}} 1 + (1/N)(\bar{L}_n/\bar{L}) \sum_{\substack{\text{nights with}\\ \text{backup}}} (Q_u/L_n) \qquad (1.2.2)$$

where N is the number of days in the heating season, and $\overline{L}d$ and $\overline{L}n$ are the average daytime and night time loads, respectively. $\overline{L}_d / \overline{L}$ is the fraction of daytime load out of heating season, daily load $\overline{L}$ and $\overline{L}_n / \overline{L}$ is the fraction of night time load out of heating season daily load. The useful energy provided by the passive solar element, denoted by Qu in eqn. (1.2.2), is a combination of both energy supplied by solar gains on the day it is collected and "residual" energy which has remained in storage from previous days. $SHF_d^{(0)}$ & $SHF_n^{(0)}$ is then used to denote SHF values for daytime and night time periods respectively, during which back-up is required, namely

$$SHF_d^{(0)} = Q_u\,(day) \,/\, \overline{L}_d \qquad (1 \cdot 2 \cdot 3)$$

$$SHF_n^{(0)} = Q_u\,(night) \,/\, \overline{L}_n \qquad (1 \cdot 2 \cdot 4)$$

In order to compute the summations in equation 1.2.2. the frequency of occurrence of diffferent values of SLR during the heating season is needed. A normalised distribution function, ρ(SLR), which gives the probability of occurrence of any SLR value is therefore introduced. SHF then becomes

$$SHF = (\overline{L}_d / \overline{L}) \left[\int_{SLR_{min,d}}^{\infty} 1 \cdot \rho(x)\,dx + \int_0^{SLR_{min,d}} \rho(x)\, SHF_d^{(0)}(x)\,dx \right]$$

$$+ (\overline{L}_n / \overline{L}) \left[\int_{SLR_{min,n}}^{\infty} 1 \cdot \rho(x)\,dx + \int_0^{SLR_{min,n}} \rho(x)\, SHF_n^{(0)}(x)\,dx \right]$$

$$(1 \cdot 2 \cdot 5)$$

The first and third terms of equation (1.2.5), represents the contribution to SHF by days and nights with no back-up, and the second and fourth terms the contribition by days and nights with back-up respectively. $SLR_{min,d}$ and $SLR_{min,n}$ are the minimum values of SLR such that no back-up heating is required during daytime and night time respectively, and the dummy variable x denotes SLR•$SLR_{min,d}$ and$_{(o)}$ $SLR_{min,n}$ are the points at which $SHF_d^{(o)}$ and $SHF_n^{(o)}$ become unity, defined using term 1 and 3 of equation (1.2.5).

$$SHF_d^{(o)}(SLR_{min,d}) = 1 \qquad (1.2.6)$$

$$SHF_n^{(o)}(SLR_{min,n}) = 1 \qquad (1.2.7)$$

It must be noted that, as shown below, in the determination of $SHF^{(o)}$ and SLR_{min}, residual energy that remains in storage from previous days, as well as energy collected on a given day, are taken into account.

1.3 The Distribution Function

In order to calculate SHF, it is required to know the function which describes the frequency of occurrence of different values of the ratio of daily insolation to load (SLR) over the course of a heating season. It should be noted that most commonly considered passive solar heated elements are vertical, so the insolation considered is that on a vertical surface. It is interesting to note that although insolation on the horizontal varies significantly during the heating season and gives rise to a skewed distribution (10,11), the insolation on the vertical is much more uniform throughout the heating season and yields far more symmetric distribution (10,11). With typical meterological years data for the coastal region of Israel (11), Gordon and Zarmi (1) determined the actual SLR distribution (assuming a thermostat set temperature of 18.3°C (65°F)). The distribution is a peaked, symmetric function with a statistical insignificent tail for large SLR values.

For the purpose of presenting a closed-form analytic solution, Gordon and Zarmi (1), used a parabolic distribution function which is peaked about the average SLR value, $\overline{SLR}$, which vanishes at $\overline{SLR} \pm \delta$ and which is normalised.

$$\rho(SLR) = \begin{cases} 0 & 0 < SLR < \overline{SLR} - \delta \\ \dfrac{3(\delta^2 - (SLR - \overline{SLR})^2)}{4\delta^3} & \overline{SLR} - \delta < SLR < \overline{SLR} + \delta \\ 0 & SLR > \overline{SLR} + \delta \end{cases} \qquad (1.3.1)$$

For the particular case cited above (11), $\delta = (0.8)\,\overline{SLR}$ (see Fig.1). The ρ(SLR) employed in equation (1 .2,5) should be the actual measured distribution function particular to ones location. In this study the data used in (1) for δ is employed. Inspection of some available data (10) indicates that $\delta / \overline{SLR}$ typically varies from 0.6 to 1.0. Also,

$$SHF_d^{(0)} = C_0^d \, SLR \qquad (1.3.2)$$

$$SHF_n^{(0)} = C_0^n \, SLR \qquad (1.3.3)$$

Since $SHF_d^{(0)}$ and $SHF_n^{(0)}$ of equation (1.2.5) equals unity at values of SLR min,d and SLR min,n respectively, i.e. equation 1.2.6 and 1.2.7, eqns (1.3.2 & 1.3.3) can be written as

$$SLR_{min,d} = 1/C_0^d \qquad (1.3.4)$$

$$SLR_{min,n} = 1/C_0^n \qquad (1.3.5)$$

Given this result, the distribution function of eqn. (1.3.1) to (1.3.3) are used in equation (1.2.5) to yield (1.3.6) below; and all terms in eqn. (1.2.5) are known for various SLRs.

$$SHF = (\bar{L}_d/\bar{L})\, SHF^{day} + (\bar{L}_n/\bar{L})\, SHF^{night} \qquad (1.3.6)$$

with

$$SHF^{day} = \begin{cases} C_0^d\, \overline{SLR} & SLR_{min,d} > \overline{SLR} + \delta \\ 1 + \left[\dfrac{(1 - C_0^d\overline{SLR} + C_0^d\delta)^3(1 - C_0^d\overline{SLR} - 3C_0^d\delta)}{16\,(C_0^d\delta)^3}\right] & \overline{SLR} + \delta \geq SLR_{min,d} \geq \overline{SLR} - \delta \\ 1 & SLR_{min,d} < \overline{SLR} - \delta \end{cases} \qquad (1.3.7)$$

$$SHF^{night} = \begin{cases} C_0^n\, \overline{SLR} & SLR_{min,n} > \overline{SLR} + \delta \\ 1 + \left[\dfrac{(1 - C_0^n\overline{SLR} + C_0^n\delta)^3(1 - C_0^n\overline{SLR} - 3C_0^n\delta)}{16\,(C_0^n\delta)^3}\right] & \overline{SLR} + \delta \geq SLR_{min,n} \geq \overline{SLR} - \delta \\ 1 & SLR_{min,n} < \overline{SLR} - \delta \end{cases} \qquad (1.3.8)$$

1.4 Application to the Direct Gain House

By definition a direct gain house is one with a large south-facing glazing and thermal storage mass in the interior. It is assumed that the building envelope is sufficiently well insulated and that storage is located so that heat transfer from storage to ambient can be neglected, and that the other thermal mass of the building is negligible relative to the storage mass

The direct gain house is treated as a two mode model, the two modes being the storage S and room air A. As all that is needed to complete the computation of SHF (via equation (1.2.5), are $SHF_d^{(0)}$ and $SHF_n^{(0)}$ (namely, the SHF values for periods during which back-up is required), the heat balance equations for the storage are considered. The rate of change of energy in storage at daytime is

$$(mC)_s\, dT_s^d / dt^d = \frac{(1-F)\,Q_T}{\Delta t^d} - U_{sA}\,(T_s^d - T_c) \qquad (1.3.9)$$

The second item of the R.H.S. of eqn. 1.3.9 represents the losses from storage S to room air at temperature, Tc of 18.3°C; $(MC)_S$ is the heat content of the storage; Ts is the storage temperature, t denotes time; subscript d denotes daytime; Δt^d denotes length of daytime; (1 - F) is fraction of transmitted insolation Q_T absorbed by storage and does not heat room air; and U_{SA} is the heat transfer coefficient from storage to room air. The solution of eqn. (1.3.9) for $T_s^d(t^d)$ is shown in appendix 1 to be

$$T_s^d(t^d) - T_c = \left[\frac{(1-F)\,Q_T(1 - e^{-t^d/\tau})}{U_{SA}\,\Delta t^d} \right] + (T_s(0) - T_c)e^{-t^d/\tau} \tag{1.3.10}$$

where

$$\tau = (mC)_s / U_{SA} \tag{1.3.11}$$

and is the relaxation time for heat transfer from storage to room air; and $T_s(0)$ is the value of T_s^d at the beginning of the day. Since no solar gain is absorbed in wall during night time, the heat balance equation for the storage at night when back-up is required is

$$(mC)_s\, dT_s^n / dt^n = 0 - U_{SA}(T_s^n - T_c) \tag{1.3.12}$$

whose solution for T_s^n is

$$T_s^n(t^n) - T_c = (T_s^d(\Delta t^d) - T_c)e^{-t^n/\tau} \tag{1.3.13}$$

where $T_s^d(\Delta t^d)$ is the value of Ts at the beginning of night time, and superscript n denotes night time. We use eqn. (1.3.10) in (1.3.13) to obtain the solution for $T_s^n(t^n)$

$$T_s^n(t^n) - T_c = \left[\left[\frac{(1-F)\,Q_T(1 - e^{-\Delta t^d/\tau})}{U_{SA}\,\Delta t^d} \right] + (T_s(0) - T_c)e^{-\Delta t^d/\tau} \right] e^{-t^n/\tau} \tag{1.3.14}$$

The relevance of the degree days, DD, is the fact that it forms a term in the calculation of the solar heating load $\bar{L}$.

The energy collected by room air during day time $Q_d^{(1)}$ and night time $Q_n^{(1)}$ on a given twenty-four hour day is

$$Q_d^{(1)} = F Q_T + \int_0^{\Delta t^d} u_{SA} \left(T_S^d(t^d) - T_c \right) dt^d \qquad (1.3.15)$$

$$Q_n^{(1)} = 0 + \int_0^{\Delta t^n} u_{SA} \left(T_S^n(t^n) - T_c \right) dt^n \qquad (1.3.16)$$

where FQ_T in eqn. 1.3.15 represents the fraction of transmitted radiation absorbed directly into room air during daytime. Using eqn.(1.3.10) and (1.3.14) in eqns (1.3.15) and 1.3.16) respectively, gives

$$Q_d^{(1)} = Q_T \left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})}{\Delta t^d} \right] \right] +$$

$$U_{SA}(T_s(0) - T_c)\tau(1-e^{-\Delta t^d/\tau}) \qquad (1.3.17)$$

and

$$Q_n^{(1)} = \frac{(1-F)Q_T\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d} +$$

$$U_{SA}(T_s(0) - T_c)\tau e^{-\Delta t^d/\tau}(1-e^{-\Delta t^n/\tau}) \qquad (1.3.18)$$

See appendices 6 and 7.
Both of equations (1.3.17) and (1.3.18) have terms which represent the fraction of Q_T which is put to use the same day and the fraction of "residual" energy (which has remained in storage from previous days). which is also put to use that same day. For a monthly or seasonal calculation, interest is of how the total useful Q_T is distributed between daytime and night time over a heating season.

Equations (1.3.17) and (1.3.18) indicate how the residual energy will, on the average, be distributed between daytime and night time. Residual energy Q_R in storage, is simply that fraction of Q_T which is not put to use on that same day, in heating room air; at night time or daytime.

$$Q_R = Q_T - (Q_d^{(1)} + Q_n^{(1)}) \qquad (1.3.19)$$

where $Q_d^{(1)}$ and $Q_n^{(1)}$ are the proportions of Q_T used in heating room air during daytime and night time respectively using the first terms of equations (1.3.17) and (1.3.18) which are the fractions of Q_T used in heating room air in eqn. (1.3.19) yield Q_R, eqn. (1.3.20) below. It should be noted that the second terms of eqn. 1. 3.17 and 1. 3.18 are the losses from storage to room air at daytime and night time periods respectively.

Hence, $Q_R = Q_T - (Q_d^{(1)} + Q_n^{(1)})$ i.e. eqn. (1.3.19). Using both eqn.(1.3.17) and (1.3.18) in (1.3.19) yields

$$Q_R = Q_T - Q_T + \frac{Q_T(1-F)\tau(1-e^{-\Delta t^d/\tau})}{\Delta t^d} - \frac{(1-F)Q_T\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d}$$

$$Q_R = \frac{Q_T(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-(1-e^{-\Delta t^n/\tau}))}{\Delta t^d}$$

$$Q_R = \frac{Q_T(1-F)\tau e^{-\Delta t^n/\tau}(1-e^{-\Delta t^d/\tau})}{\Delta t^d} \qquad (1.3.20)$$

Consequently, the average useful daytime (night time) energy gain Q_d (Q_n) for days with no residual energy $Q_R = 0$ (or days which require backing up) is obtained as (see Appendix 8).

$$Q_d = Q_T\left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d(1-e^{-(\Delta t^d+\Delta t^n)/\tau}}\right]\right]$$

i.e

$$Q_T = Q_d + Q_n + Q_R(0) \qquad \text{OR} \qquad Q_n = Q_T - Q_d \qquad (1.3.21)$$

Equation (1.3.21) provides the solution for SHF (Q_d/Q_T) for days during which back-up is required ($Q_R = 0$) namely

$$SHF_{day} = \frac{Q_d}{Q_T} = \left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d(1-e^{-(\Delta t^d+\Delta t^n)/\tau})}\right]\right]$$

(1.3.22)

By defination

$$SHF_{day} = C_o^d\, SLR \qquad (1.3.23)$$

Hence the RHS of eqn (1.3.22) equals the RHS of eqn (1.3.23) OR

$$C_o^d(\bar{L}_d/\bar{L}) = \left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d(1-e^{-(\Delta t^d+\Delta t^n)/\tau})}\right]\right]$$

Therefore

$$C_o^d = \left(\frac{\bar{L}}{\bar{L}_d}\right)\left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d(1-e^{-(\Delta t^d+\Delta t^n)/\tau})}\right]\right] \quad (1.3.24)$$

It can be similarly shown that

$$C_o^n = \left(\frac{\bar{L}}{\bar{L}_n}\right)\left[\frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d(1-e^{-(\Delta t^d+\Delta t^n)/\tau})}\right] \quad (1.3.25)$$

by writing $Q_n = Q_T - Q_d$ in eqn. (1.3.21) and following the same procedure.

The fact that $SHF^{(0)}$ (days with back-up, $Q_R = 0$) is proportional to SLR is a key result. It enables one to perform the integration described in section (1.2), and to obtain a closed form analytic expression for SHF. With these results, eqns (1.3.22)-(1.3.25) and eqn.(1.3.1) are used in eqn (1.2.5) to yeild the closed-form expressions for SHF as described in eqns (1.3.7)-(1.3.8).

It has been assumed above that insolation can effectively be treated as constant throughout the day. The justification for this is presented below. The only change on considering the effect of using a time-dependent insolation during daytime is the replacement of the constant insolation rate $Q_d/\Delta t^d$ used in daytime eqn. (1.3.9) by a time-dependent one.

The gross feature of the time dependence of the insolation can be represented (12) by a function of the form

$$q_o(\cos w - \cos w_s) \qquad (1\cdot3\cdot26)$$

where w_s is the sunrise(sunset hour angle.

$$w_s = \pi \Delta t^d / T \qquad (T = \Delta t^d + \Delta t^n = 24\,hr) \qquad (1\cdot3\cdot27)$$

and

$$w = w_s - \Omega t \qquad (\Omega = 2\pi / T) \qquad (1\cdot3\cdot28)$$

Eqn (1·3·9) now becomes

$$(mC)_s dT_s^d / dt^d = (1-F) q_o (\cos w - \cos w_s) - U_{SA}(T_s^d - T_c) \qquad (1\cdot3\cdot29)$$

The solution of eqn (1·3·29) yields

$$T_s^d - T_c = \frac{(1-F)q_o}{U_{SA}} \left[\left[\left[\frac{\Omega \tau \sin w_s - \cos w_s}{1 + \Omega^2 \tau^2} \right] + \cos w_s \right] e^{-t^d/\tau} + \left[\frac{\cos w - \Omega \tau \sin w}{1 + \Omega^2 \tau^2} \right] - \cos w_s \right] + (T_s^d(0) - T_c) e^{-t^d/\tau}$$

$$(1\cdot3\cdot30)$$

Repeating the procedure of section (1.4), $Q_d^{(1)}$ $Q_n^{(1)}$ and Q_R are recalculated. The partition of the residual energy Q_R between daytime and night time is still the same as in section (1.4), yielding the final partition of Q_T between daytime and night-time as

$$Q_n = \frac{(1-F)\,Q_T\,\Omega^2\tau^2}{2(1+\Omega^2\tau^2)}\left[\frac{\sin\omega_s - \Omega\tau\cos\omega_s + (\sin\omega_s + \Omega\tau\cos\omega_s)e^{-\Delta t^d/\tau}}{\sin\omega_s - \omega_s\cos\omega_s}\right]$$

$$\left[\frac{1-e^{-\Delta t^n/\tau}}{1-e^{-T/\tau}}\right] \qquad (1.3.31)$$

$$Q_d = Q_T - Q_n$$

These expressions for Q_n and Q_d are clearly different from the corresponding ones calculated in section 1.4 (see eqn(1.3.21)). The main difference appears at low values of τ, where eqn (1.3.21) yields

$$Q_n = (1-F)\,Q_T\,\tau/\Delta t^d \qquad (1.3.32)$$

$$Q_n = (1-F)\,Q_T\left[\frac{\sin\omega_s}{2(\sin\omega_s - \omega_s\cos\omega_s)}\right]\Omega^2\tau^2 \qquad (1.3.33)$$

For commonly used values of, τ, however, the difference between the two results is negligible. For example, for a Δt^d of 10 hr and a value of τ as small as 5 hr, eqn (1.3.31) yields for Q_n a value which is smaller than that obtained by eqn (1.3.21) by 6.4%. With increasing τ, agreement is better, and for large values of τ both expressions for Q_n approach the same limit.

$$Q_n \xrightarrow[\text{Large } \tau]{} (1-F)\,Q_T\,\Delta t^n/T \qquad (1.3.34)$$

Also, the limit of Q_n for short daytime periods (Δt^d small) is the same for both calculations.

$$Q_n \xrightarrow[\Delta t^d \text{ small}]{} (1-F)\,Q_T \qquad (1\cdot3\cdot35)$$

1.5 Application to the Water Wall House

The house to be considered has a glazed, south-facing water storage wall. The absorbing surface of the water wall is taken to be equal to the glazing area. The houses's other thermal mass is assumed to be negligible relative to the water wall. The use of night insulation is optional and will be treated in detail below.

The wall house is treated with a two-mode model, the two nodes being the water storage S and warm air A. During daytime when back-up is required, the heat transfer rate from storage to room air is represented by

$$(mc)_s \, dT_s^d / dt^d = (\alpha Q_T / \Delta t^d) - U_{SA}(T_s^d - T_a^d) - U_{SA}(T_s^d - T_c) \qquad (1\cdot5\cdot1)$$

where α is the fraction of transmitted insolation Q_T which is absorbed by storage; U_{sa}^d is the daytime heat transfer coefficient from storage to ambient; and U_{SA} is the heat transfer coefficient from storage to room air.

One further approximation to be made here is that the relative fast rate of heat transfer which results from natural convection within the water wall allows the treatment of the water wall as an element of negligible thermal resistance. The convective heat transfer coefficient within a water wall can in fact be calculated from the theory of natural convection of a fluid between two parallel plates (13).

For the temperature differences typically encountered in such systems, the thermal resistance to heat flow by the water wall is of the order of 0 - 5% of the thermal resistance to heat flow from the inner surface of the water wall to room air and will hence be treated as a negligible contribution to the overall thermal resistance to heat flow from storage to room air, $1 / u_{sA}$

The solution of eqn (1.5.1) for $T_s^{\,d}(t^d)$ is shown in appendix 2 to be

$$T_s^{\,d}(t^d) - T_c = \left[\frac{\alpha Q_T(1-e^{-t^d/\tau^d})}{\Delta t^d(u_{sA}+u_{sa}^d)}\right] - \left[\frac{u_{sa}^d(T_c - T_a^d)(1-e^{-t^d/\tau^d})}{u_{sA}+u_{sa}^d}\right]$$

where

$$+ (T_s(0) - T_c)e^{-t^d/\tau^d} \qquad (1.5.2)$$

$$\tau^d = (mC)_s / (u_{sA} + u_{sa}^d) \qquad (1.5.3)$$

and is the relaxation time for daytime heat transfer from storage to both room air and ambient; and $T_s(0)$ is the value of $T_s^{\,d}$ at the beginning of the day. At night time when no energy is absorbed by storage, the heat balance equation when back-up is required is

$$(mC)_s\, dT_s^{\,n}/dt^n = 0 - u_{sa}^n(T_s^{\,n} - T_a^n) - u_{sA}(T_s^{\,n} - T_c)$$

$$(1.5.4)$$

whose solution for $T_s^{\,n}$ is

$$T_s^{\,n}(t^n) - T_c = \left[\frac{-u_{sa}^n(T_c - T_a^n)(1-e^{-t^n/\tau^n})}{u_{sA}+u_{sa}^n}\right]$$

$$+ (T_s^{\,d}(\Delta t^d) - T_c)e^{-t^n/\tau^n} \qquad (1.5.5)$$

where $T_s^d(\Delta t^d)$ is the value of T_s at the beginning of nightime, superscript n

denotes night time; and

$$\tau^n = (mC)_s / (U_{sA} + U_{sa}^n) \qquad (1.5.6)$$

A distinction is made between the daytime and night time heat transfer coefficients from storage to ambient (U_{sa}^d and U_{sa}^n) in order to allow for the effect of night insulation, should it be employed. Using eqn (1.5.2) in (1.5.5) yields the solution for $T_s^n(t^n)$,

$$T_s^n(t^n) - T_c = \left[\frac{\alpha Q_T (1 - e^{-\Delta t^d/\tau^d})\, e^{-t^n/\tau^n}}{\Delta t^d (U_{sA} + U_{sa}^d)} \right]$$

$$- \left[\frac{U_{sa}^d (T_c - T_a^d)(1 - e^{-\Delta t^d/\tau^d})\, e^{-t^n/\tau^n}}{U_{sA} + U_{sa}^d} \right]$$

$$- \left[\frac{U_{sa}^n (T_c - T_a^n)(1 - e^{-t^n/\tau^n})}{U_{sA} + U_{sa}^n} \right]$$

$$+ \ (T_s(0) - T_c)\, e^{-\Delta t^d/\tau^d})\, e^{-t^n/\tau^n} \qquad (1.5.7)$$

The energy collected by room air during daytime ($Q_d^{(1)}$) and night time ($Q_n^{(1)}$) on a given twenty-four hour day is

$$Q_d^{(1)} = \int_0^{\Delta t^d} U_{sA} (T_s^d(t^d) - T_c)\, dt^d \qquad (1.5.8)$$

$$Q_n^{(1)} = \int_0^{\Delta t^n} U_{sA} (T_s^n(t^n) - T_c)\, dt^n \qquad (1.5.9)$$

Similarly, the energy lost from storage to ambient during daytime ($E_d^{(1)}$) and night time ($E_n^{(1)}$) for the same twenty-four hour day is

$$E_d^{(1)} = \int_0^{\Delta t^d} U_{sa}^d \left(T_s^d(t^d) - T_a^d \right) dt^d \qquad (1{\cdot}5{\cdot}10)$$

$$E_n^{(1)} = \int_0^{\Delta t^n} U_{sa}^n \left(T_s^n(t^n) - T_a^n \right) dt^n \qquad (1{\cdot}5{\cdot}11)$$

Using eqn (1.5.2) and (1.5.7) in eqns (1.5.8)-(1.5.11), gives

$$Q_d^{(1)} = \frac{\alpha Q_T U_{SA}}{(U_{SA} + U_{Sa}^d)} \left[1 - \left[\frac{\tau^d (1 - e^{-\Delta t^d/\tau^d})}{\Delta t^d} \right] \right] \quad \text{gain}$$

$$+ \quad U_{SA} \tau^d (T_s(0) - T_c)(1 - e^{-\Delta t^d/\tau^d}) \quad \text{residual}$$

$$- \quad L_1 \quad \text{wall Load}$$

$$(1{\cdot}5{\cdot}12)$$

$$Q_n^{(1)} = \frac{\alpha Q_T U_{SA} \tau^n (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})}{\Delta t^d (U_{SA} + U_{Sa}^d)} \quad \text{gain}$$

$$+ \quad U_{SA} \tau^n (T_s(0) - T_c) e^{-\Delta t^d/\tau^d} (1 - e^{-\Delta t^n/\tau^n})$$

residual

$$- \quad L_2 \quad \text{wall Load}$$

$$(1{\cdot}5{\cdot}13)$$

$$E_d^{(1)} = \frac{\alpha Q_r U_{sa}^d}{(U_{sA} + U_{sa}^d)} \left[1 - \left[\frac{\tau^d (1 - e^{-\Delta t^d/\tau^d})}{\Delta t^d} \right]^{\cdot} \right]$$

gain

$$+ \quad U_{sa}^d \tau^d (T_s(0) - T_c)(1 - e^{-\Delta t^d/\tau^d})$$

residual

$$+ \quad L_1$$

wall Load

$$(1{\cdot}5{\cdot}14)$$

$$E_n^{(1)} = \frac{\alpha Q_r U_{sa}^n \tau^n (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})}{\Delta t^d (U_{sA} + U_{sa}^d)}$$

gain

$$+ \quad U_{sa}^n \tau^n (T_s(0) - T_c) e^{-\Delta t^d/\tau^d} (1 - e^{-\Delta t^n/\tau^n})$$

residual

$$+ \quad L_3$$

wall Load

$$(1{\cdot}5{\cdot}15)$$

Eqn.(1.5.12)-(1.5.15) each have three types of terms. Terms proportional to αQ_T represent the fraction of the absorbed solar gain, αQ_T , which is transferred either to room air ($Q_d^{(1)}$ $ $Q_n^{(1)}$) or to ambient ($E_d^{(1)}$) and ($E_n^{(1)}$) on the same day. Terms proportional to $T_s(o) - T_c$ represent the fraction of "residual" energy which has remained in storage from previous days) which is also transferred to either room air or ambient that same day. Terms L_1, L_2 and L_3 represent the contributions of the water wall to the heating load on that same day, and will not enter any further into computations. Hence their exact form is omitted for the sake of brevity. It is of interest to obtain the distribution of the total useful solar gain between daytime and night time periods over the course of a heating season. Equations (1.5.12)-(1.5.15) indicate how the transfer of residual energy to either room air or ambient will, on the average, be distributed between daytime and night time. Residual energy is simply that fraction of αQ_T which is not transferred to either room air or ambient on that same day. From eqns (1.5.12)-(1.5.15) Q_R is obtained as

$$Q_R = \frac{\alpha Q_T \tau^d}{\Delta t^d} (1 - e^{-\Delta t^d/\tau^d}) e^{-\Delta t^n/\tau^n} \tag{1.5.16}$$

Consequently, the average useful daytime (night time) energy gain to room air Q_d (Q_n) is

$$Q_d = \frac{\alpha Q_T \tau U_{SA}}{U_{SA} + U_{Sa}^d} \left[1 - \left[\frac{\tau^d (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})}{} \right] \right] \tag{1.5.17}$$

$$Q_n = \frac{\alpha Q_T U_{SA} \tau^d (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})}{\Delta t^n (U_{SA} + U_{Sa}^n)(1 - e^{-(\Delta t^d/\tau^d + \Delta t^n/\tau^n)})} \tag{1.5.18}$$

Eqns (1.5.17) and (1.5.18) provide the solution for the SHF for days during which back-up is required, namely

$$SHF_d^{(o)} = Q_d / \bar{L}_d = \left(\frac{\bar{L}}{\bar{L}_d} \right) (Q_d / \bar{L}) = C_o^d \, SLR \quad (1.5.19)$$

and

$$SHF_n^{(o)} = Q_n / \bar{L}_n = (\bar{L}/\bar{L}_n) (Q_n / \bar{L}) = C_o^n \, SLR \quad (1.5.20)$$

where $C_o^d = (\bar{L}/\bar{L}_d) \dfrac{\alpha U_{SA}}{U_{SA} + U_{Sa}^d} \left[1 - \left[\dfrac{\tau^d (1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})}{\Delta t \,(1-e^{-(\Delta t^d/\tau^d + \Delta t^n/\tau^n)})} \right] \right]$ (1.5.21)

$$C_o^n = \frac{(\bar{L}/\bar{L}_n)\, \alpha \, U_{SA} \, \tau^d}{\Delta t^d (U_{SA} + U_{Sa}^n)} \left[\frac{(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})}{1 - e^{-(\Delta t^d/\tau^d + \Delta t^n/\tau^n)}} \right] \quad (1.5.22)$$

The key result is the fact that $SHF^{(o)}$ is proportional to SLR and the closed-form expressions for SHF are eqns (1.3.6) - (1.3.8).

1.6 Application to the house with Massive Storage or Trombe Wall

The house to be considered here has a glazed, south-facing massive storage wall (MSW). The absorbing surface area of the MSW is taken to be equal to the glazing area. The house's other thermal mass is assumed to be negligible relative to the MSW. The use of night insulation is optional and affects the value of U_{sa}^n, The night time heat transfer coefficient from storage to ambient. Treatment is made of the two cases of vented and unvented MSWs.

The heat balance equation for the storage is considered with the storage represented by a single temperature T_S . The validity of treating the non-linear problem of heat diffusion through the MSW by a linear one node model, for the calculation of long term thermal performance for wall thickness of practical interest, is established in (14). See Appendix 3.

Based on the assumption that the non-linear solution for MSW temperature T_s does not introduce much difference to the one-node model (14), during the daytime when back-up is required, the heat balance equation for storage is

$$(mc)_s \, dT_s^d / dt^d = \left[\alpha Q_T / \Delta t^d \right] - U_{sa}^d (T_s^d - T_a^d)$$
$$- U_w^d (T_s^d - T_c) \qquad (1.6.1)$$

where $(MC)_s$ is the heat content of the storage; superscript d denotes daytime, T_a^d is the average daytime ambient temperature; t denotes time; Δt^d is the length of day time; α is the fraction of transmitted insolation Q_T which is absorbed by storage; U_{sa}^d is the daytime heat transfer coefficient from storage to ambient; and U_w^d is the daytime heat transfer coefficient from storage to room air, given by

$$U_w^d = \begin{cases} U_{sw} & \text{unvented MSW} \\ U_{sw} + U_{vent} & \text{vented MSW} \end{cases} \qquad (1.6.2)$$

In equation 1.6.2, U_{sw} is the heat transfer coefficient from storage to room air, through the internal storage surface, and U_{vent} is an effective heat transfer coefficient which approximately represents the effect of venting.

A fairly accurate accounting for venting on long time scales (a month or longer) by introducing one effective value of U_{vent} has been demonstrated by computer simulations of vented MSWs (15). The justification for the treatment of insolation as a constant has been presented in section 1.4. For the one-node model, U_{sw} is given by

$$U_{sw} = 1/((1/U_{SA}) + (L/K))$$
$$(1.6.3)$$

where U_{SA} is the surface-to-air heat transfer coefficient, L is the wall thickness, and K is the thermal conductivity of the MSW material.

The solution of eqn.(1.6.1) for $T_s^d(t^d)$ is

$$T_s^d(t^d) - T_c = \left[\frac{\alpha Q_T(1-e^{-t^d/\tau^d})}{\Delta t^d(U_w^d + U_{sa}^d)}\right] - \left[\frac{U_{sa}^d(T_c - T_a^d)(1-e^{-t^d/\tau^d})}{(U_w^d + U_{sa}^d)}\right]$$

$$+ (T_s(0) - T_c)e^{-t^d/\tau^d} \qquad (1.6.4)$$

where

$$\tau^d = (mC)_s / (U_w^d + U_{sa}^d) \qquad (1.6.5)$$

and is the relaxation time for daytime heat transfer from storage to both room air and ambient; and $T_s(0)$ is the value of T_s^d at the beginning of the day.

The heat balance equation for the storage at night when back-up is required is

$$(mC)_s\, dT_s^n/dt^n = 0 - U_{sa}^n(T_s^n - T_a^n) - U_{sw}(T_s^n - T_c) \qquad (1.6.6)$$

whose solution for T_s^n is

$$T_s^n(t^n) - T_c = -\frac{U_{sa}^n(T_c - T_a^n)(1-e^{-t^n/\tau^n})}{(U_{sw} + U_{sa}^n)}$$

$$+ (T_s^d(\Delta t^d) - T_c)e^{-t^n/\tau^n}$$

$$(1.6.7)$$

where $T_s^d(\Delta t^d)$ is the value of T_s at the beginning of night time; superscript n denotes night time; and

$$\tau^n = (mC)_s / (U_{sw} + U_{sa}^n) \qquad (1.6.8)$$

The distinction between U_{sa}^d and U_{sa}^n is to allow for the effect of night insulation should it be employed. Using eqn (1.6.4) in (1.6.7) gives the solution for $T_s^n(t^n)$.

$$\begin{aligned} T_s^n(t^n) - T_c = & \left[\alpha Q_T (1 - e^{-\Delta t^d/\tau^d}) e^{-t^n/\tau^n} \right] \\ & - \left[\frac{U_{sa}^n (T_c - T_a^n)(1 - e^{-t^n/\tau^n})}{(U_{sw} + U_{sa}^n)} \right] \\ & - \left[\frac{U_{sa}^d (T_c - T_a^d)(1 - e^{-\Delta t^d/\tau^d}) e^{-t^n/\tau^n}}{(U_w^d + U_{sa}^d)} \right] \\ & + (T_s(0) - T_c) e^{-\Delta t^d/\tau^d} e^{-t^n/\tau^n} \end{aligned} \qquad (1.6.9)$$

The energy collected by storage on a given 24 hour day is composed of three components: (a) heat lost to ambient; (b) heat transferred to room air; and (c) "residual" heat which remains in storage and is consumed on subsequent days. Following the procedure detailed in section (1.4), the SHF for days during which backup is required is obtained as

$$SHF_d^{(0)} = C_0^d \cdot SLR \qquad (1.6.10)$$

$$SHF_n^{(0)} = C_0^n \cdot SLR \qquad (1.6.11)$$

where

$$C_0^d = (\bar{L}/\bar{L}_d) \frac{\alpha U_w^d}{(U_w^d + U_{sa}^d)} \left[1 - \left[\frac{\tau^d (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})}{\Delta t^d (1 - e^{-(\Delta t^d/\tau^d + \Delta t^n/\tau^n)})} \right] \right] \qquad (1.6.12)$$

$$C_0^n = (\bar{L}/\bar{L}_n) \frac{U_{sw}}{(U_{sw} + U_{sa}^n)} \left[\frac{\tau^d (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})}{\Delta t^d (1 - e^{-(\Delta t^d/\tau^d + \Delta t^n/\tau^n)})} \right] \qquad (1.6.13)$$

The final expression for SHF is

$$SHF = SHF^{day}(\bar{L}_d / \bar{L}) + SHF^{night}(\bar{L}_n / \bar{L}) \quad (1.6.14)$$

where SHF^{day} and SHF^{night} are the average daytime and night time SHFs respectively. If the parabolic distribution function described in section 2 is now employed, a closed analytic expression for SHF is obtained, with

$$SHF^{day} = \begin{cases} C_o^d x_A & SLR_{min,d} > x_A + \delta \\ 1 + \dfrac{(1 - C_o^d x_A + C_o^d \delta)^3 (1 - C_o^d x_A - 3C_o^d \delta)}{16(C_o^d \delta)^3} & x_A + \delta \geqslant SLR_{min,d} \geqslant x_A - \delta \\ 1 & SLR_{min,d} < x_A - \delta \end{cases} \quad (1.6.15)$$

$$SHF^{night} = \begin{cases} C_o^n x_A & SLR_{min,n} > x_A + \delta \\ 1 + \dfrac{(1 - C_o^n x_A + C_o^n \delta)^3 (1 - C_o^n x_A - 3C_o^n \delta)}{16(C_o^n \delta)^3} & x_A + \delta \geqslant SLR_{min,n} \geqslant x_A - \delta \\ 1 & SLR_{min,n} < x_A - \delta \end{cases} \quad (1.6.16)$$

where x_A = average SLR value.

1.7 Application to the Direct Gain House with An Attached Conservatory

In keeping with the original work of Gordon and Zarmi, we consider the relative proportions of the size of the conservatory to the direct gain system, where B is the quantitative heat transfer proportion of air from the direct gain system to the conservatory. Then a study can be made of the effect of factors as the size of the conservatory on the direct gain system. It is further assumed that the complete outward wall surface area (south facing) between the direct gain system and the conservatory constitutes the system storage mass. The direct gain system is considered at temperature T_c . As pointed out in section (1.4) the effect of considering a constant insolation rate Q_T on the computations can be assumed negligible. The storage is considered at temperature $T_s^{\,d}$ during daytime, and $T_s^{\,n}$ at night time. T_{co} is the average uniform temperature of the conservatory, and $T_a^{\,d}$ the average daytime ambient temperature. A consideration of SHF on SLR for such a system and the influence on it of the building and climatic factors will be made. Evaluations of the performance of such a system as opposed to the direct gain house considered by Gordon and Zarmi (1) will be presented in section (2) below.

The daytime heat balance equation for this system considering a 3 node model is

$$(mC)_s\, dT_s^{\,d}/dt^d = (1-B)\left[((1-F)Q_T/\Delta t^d) - U_{SA}(T_s^{\,d}-T_c)\right] + B\left[U_{co,s}(T_{co}^{\,d}-T_s^{\,d})\right] + B\left[\frac{(1-F_c)Q_T}{\Delta t^d}\right] \qquad (1\cdot7\cdot1)$$

where $(mC)_s$ is the heat content of the storage ; superscript d denotes day time; Δt^d is the length of daytime; F is the fraction of transmitted insolation Q_T which is not absorbed by storage and hence directly heats room air; F_c the fraction of transmitted insolation ;through conservatory not absorbed by storage (wall & storage in conservatory), and directly heats the air in the conservatory. U_{SA} and $U_{co,s}$ represent the heat transfer coefficients from storage to room air and conservatory to storage respectively, and B the proportion of air in the conservatory relative to the direct gain system. It is assumed that the building is well insulated and at night time losses from storage, are to room air and conservatory only. It could be further assumed that the storage is located so that at night time heat transfer from storage to conservatory can be neglected. This is for a system with night time insolation.

The solution of eqn. (1.7.1) for $T_s^d(t^d)$ is shown in appendix 4 to be

$$
\begin{aligned}
T_s^d(t^d) - T_c = & \left[\frac{(1-B)(1-F)\,Q_T\,(1-e^{-t^d/\tau^d})}{\Delta t^d\,[(1-B)\,U_{SA} + B\,U_{co,s}]} \right] \\
& + \left[\frac{(1-F_c)\,Q_T \cdot B \cdot (1-e^{-t^d/\tau^d})}{[(1-B)\,U_{SA} + B\,U_{co,s}]\,\Delta t^d} \right] \\
& - \left[\frac{B \cdot U_{co,s}\,(1-e^{-t^d/\tau^d})\,(T_c - T_{co}^d)}{(1-B)\,U_{SA} + B\,U_{co,s}} \right] \\
& + (T_s(0) - T_c)\,e^{-t^d/\tau^d}
\end{aligned}
\qquad (1.7.2)
$$

or

$$T_s^d - T_c = \left[\frac{Q_T(1-e^{-t^d/\tau^d})(1-F+BF-BF_c)}{\Delta t^d \left[(1-B)U_{SA} + B\,U_{co,s}\right]} \right]$$

$$- \left[\frac{B \cdot U_{co,s}(1-e^{-t^d/\tau^d})(T_c - T_{co}{}^d)}{(1-B)U_{SA} + B\,U_{co,s}} \right]$$

$$+ (T_s(o) - T_c)e^{-t^d/\tau^d} \qquad (1.7.3)$$

where

$$\tau^d = (mc)_s / (1-B)U_{SA} + B\,U_{co,s} \qquad (1.7.4)$$

and is the relaxation time for both heat transfer from storage to room air, and conservatory to storage; $T_s(o)$ is the value of $T_s{}^d$ at the beginning of the day.

The heat balance equation for the storage at night when back-up is required is

$$(mc)_s\, dT_s{}^n / dt^n = 0 - (1-B)\left[U_{SA}(T_s{}^n - T_c)\right] \qquad (1.7.5)$$

This is the equation balancing heat transfer in the storage assuming night time insolation is employed. the first term of eqn (1.7.1) is zero. So are the third and fourth terms respectively. Equation 1.7.1 reduces to the original form due to Gordon and Zarmi (1). If however no night insolation is employed, there will be losses from storage to conservatory and the first and last terms of eqn (1.7.1) vanish. The third term, however, now becomes

Term 3 $= -B[U_{S,CO}(T_S^n - T_{CO}^n)]$ (1.7.6)

or Term 3 $= -B[U_{CO,S}(T_S^n - T_a^n)]$ (1.7.7)

i.e. assuming the situation in the conservatory represents night time, outdoor ambient conditions with allowable accuracy.

Hence, the night time heat balance equation can be written (for the case of no night time insulation) as

$$(mC)_S\, dT_S^n/dt^n = 0 - (1-B)\left[U_{SA}(T_S^n - T_c)\right] - B\left[U_{CO,S}(T_S^n - T_{CO}^n)\right] \quad (1.7.8)$$

We note that when (B = 0), the equation 1.7.8 reduces to the direct gain system without conservatory i.e. eqn. 1.3.12.,the solution of eqn (1.7.5) can be shown to be (for the case of night time insolation)

$$T_S^n(t^n) - T_c = (T_S^d(\Delta t^d) - T_c)\, e^{-t^n/\tau^n} \quad (1.7.9)$$

where

$$\tau^n = (mC)_S / (1-B)\, U_{SA} \quad (1.7.10)$$

and is the relaxation time for heat transfer from storage to room air at night time; where $T_S^d(\Delta t^d)$ is the value of T_S at the beginning of night time, and superscript n denotes night time.

Using eqn (1.7.3) in (1.7.9) gives the solution for $T_S^n(t^n)$ as

$$T_S^n(t^n) - T_c = \left[\left[\frac{Q_T(1-e^{-\Delta t^d/\tau^d})(1-F+BF-BF_c)}{\Delta t^d((1-B)U_{SA} + B\,U_{CO,S})}\right] - \left[\frac{B\cdot U_{CO,S}(1-e^{-\Delta t^d/\tau^d})(T_c - T_{CO}^d)}{(1-B)U_{SA} + B\,U_{CO,S}}\right] + (T_S(0) - T_c)e^{-\Delta t^d/\tau^d}\right] e^{-t^n/\tau^n}$$

(1.7.11)

In eqn. (1.7.11), τ^d and τ^n are given by equations (1.7.4) and (1.7.10) respectively.
It must be noted that equation 1.7.11 is the solution of eqn.(1.7.5) (i.e. using night time insolation). If no night insulation is employed, then eqn. (1.7.8) must be solved for T_s^n .

The solution for T_s^n using no night time insulation can be shown to be (i.e. solution of eqn. (1.7.8)). See appendix 5.

$$T_s^n(t^n) - T_c = -\left[\frac{B\,U_{co,s}(1-e^{-t^n/\tau^n})(T_c - T_a^n)}{(1-B)\,U_{SA} + B\,U_{co,s}}\right] + (T_s^d(\Delta t^d) - T_c)\,e^{-t^n/\tau^n} \quad (1.7.12)$$

where $T_s^d(\Delta t^d)$ is the value of T_s at the beginning of night time, and the relaxation time τ^n is given by

$$\tau^n = (mC)_s / (1-B)\,U_{SA} + B\,U_{co,s} \quad (1.7.13)$$

Using eqn.(1.7.3) in 1.7.12 gives the solution for T ( ) (with no night time insulation employed) as (Appendix 5).

$$T_s^n(t^n) - T_c = \left[\left[\frac{Q_r(1-e^{-\Delta t^d/\tau^d})(1-F+BF-BF_c)}{\Delta t^d\left[(1-B)\,U_{SA} + B\,U_{co,s}\right]}\right]\right.$$

$$- \left[\frac{B\cdot U_{co,s}(1-e^{-\Delta t^d/\tau^d})(T_c - T_{co}^d)}{(1-B)\,U_{SA} + B\,U_{co,s}}\right]$$

$$\left. + (T_s(0) - T_c)\,e^{-\Delta t^d/\tau^d}\right]e^{-t^n/\tau^n}$$

$$- \left[\frac{B\cdot U_{co,s}(1-e^{-t^n/\tau^n})(T_c - T_a^n)}{(1-B)\,U_{SA} + B\,U_{co,s}}\right]$$

$$(1.7.14)$$

where τ^d and τ^n are given by eqns. (1.7.4) and (1.7.13) respectively.

Thus equation (1.7.11) and (1.7.14) presents us with the solution for $T_s^n(t^n)$ with and without night time insulation respectively. In equation 1.7.11 the relaxation times (τ^d, τ^n) are given by equations 1.7.4 and 1.7.10 respectively. For the use of eqn. 1.7.14, τ^d, τ^n are given by eqns (1.7.4) and (1.7.13) respectively.

We should now evaluate the energy (for a given twenty-four hour day) collected by room air during daytime ($Q_d^{(1)}$) and night time ($Q_n^{(1)}$). The later will be done for both the cases of night time and no night time insulation.

$$Q_d^{(1)} = FQ_T + \int_0^{\Delta t^d} U_{SA}\left(T_s^d(t^d) - T_c\right) dt^d \qquad (1.7.15)$$

$$Q_n^{(1)} = \int_0^{\Delta t^n} U_{SA}\left(T_s^n(t^n) - T_c\right) dt^n \qquad (1.7.16)$$

where FQ_T in eqn. (1.7.15) represents the fraction of transmitted radiation absorbed directly into room air during daytime.

Using equation (1.7.3) in (1.7.15) gives (see Appendix 9).

$$Q_d^{(1)} = FQ_T \left[1 - \frac{U_{SA}(1-F+BF-BF_c)}{F\left[(1-B)U_{SA} + B\,U_{co,s}\right]} \left[\frac{\tau^d(1-e^{-\Delta t^d/\tau^d})}{\Delta t^d} - 1 \right] \right]$$

$$+ \; U_{SA}(T_s(0) - T_c)\,\tau^d(1-e^{-\Delta t^d/\tau^d}) \; - \; L_1$$

where L_1 is given in Appendix 9.

Equation (1.7.17) has three types of terms. The term proportional to FQ_T represents the fraction of the absorbed solar gain, FQ_T, which is transferred to room air. The term proportional to $T_s(0) - T_c$, represents the fraction of "residual" energy (which has remained in storage from previous days) which is also transferred to room air. The term L_1 represents the losses from room air to conservatory. The exact solution will be presented by eliminating the second and third terms of eqn (1.7.17)$(1.7.18).

Similarly, for the case where night insulation is employed, using eqn (1.7.1) in (1.7.16) yields the solution of $Q_n^{(1)}$ as (see Appendix 10).

$$Q_n^{(1)} = \frac{mFQ_T U_{SA}\tau^n(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})}{\Delta t^d\left[(1-B)U_{SA} + B\,U_{co,s}\right]}$$

$$+ \; U_{SA}\,\tau^n(T_s(0) - T_c)\,e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n})$$

$$- L_2 \qquad (1.7.18)$$

where $\quad m = \eta / F$

$$L_2 = \frac{U_{SA}\cdot B \cdot U_{co,s}\,\tau^n(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})(T_c - T_{co}^d)}{((1-B)U_{SA} + B\,U_{co,s})}$$

$$(1.7.19)$$

In equation 1.7.18, τ^d, τ^n are given by equations (1.7.4) and (1.7.10) respectively, and

$$\eta = 1 - F + BF - BF_c \qquad (1.7.20)\,a$$

In order to evaluate the energy collected by room air during night time for the direct gain system with attached conservatory and no night insulation employed; the solution for the heat balance equation, (eqn.1.7.14), is substituted in equation (1.7.16). τ^d, τ^n are now given by equations (1.7.4) and (1.7.13).

Hence,(see Appendix 11); distinguishing the energy collected by room air at night time for the direct gain system with attached conservatory, and no night time insulation as $Q_d^{(1*)}$, Appendix 11 gives

$$Q_d^{(1*)} = Q_d^{(1)} - L_3 \qquad (1.7.20)\,b$$

where
$$L_3 = \frac{U_{SA} \cdot B \cdot U_{co,s}\, \tau^n (\Delta t^n/\tau^n + e^{-\Delta t^n/\tau^n} - 1)(T_c - T_{co}^n)}{((1-B)U_{SA} + BU_{co,s})} \qquad (1.7.21)$$

As noted in Appendix 11, the effect of not employing night time insulation is to reduce the energy collected by room air by a value equal to L_3 .

Residual energy is simply that fraction of FQ_T which is not transferred to either room air or conservatory on the same day. It is evaluated using the terms in FQ_T in equations (1.7.17) and (1.7.18) for the system with night time insulation as

$$Q_R = FQ_T - (Q_d^{(1)} + Q_n^{(1)})$$

$$= FQ_T - FQ_T + \frac{mFQ_T U_{SA}}{(1-B)U_{SA} + BU_{CO,S}} \left[\frac{\tau^d(1-e^{-\Delta t^d/\tau^d})}{\Delta t^d} - 1 \right]$$

$$- \frac{mFQ_T U_{SA} \tau^n (1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})}{\Delta t^d((1-B)U_{SA} + BU_{CO,S})}$$

$$= \frac{mFQ_T U_{SA} [\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d - \tau^n(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})]}{\Delta t^d((1-B)U_{SA} + BU_{CO,S})}$$

OR

$$Q_R = \frac{mFQ_T U_{SA}}{(1-B)U_{SA} + BU_{CO,S}} \frac{[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d - \tau^n(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})]}{\Delta t^d}$$

$$(1.7.22)$$

The average useful daytime (night time) energy gain Q_d (Q_n) is evaluated by solving eqn. (1.7.17) and (1.7.18) simultaneously. Eliminating the terms in $T_s(0) - T_c$ and L_1 (L_2) yields (see Appendix 12) for the system with night insulation.

SUMMARY

From Appendix 12 to 14 we summarize as follows.

For a direct gain system with an attached conservatory, the useful energy gains $Q_d(Q_n)$ at daytime and night time are given by

(A) System with night insulation (A12)

The average useful daytime (night time) energy gain Q_d (Q_n) is

$$Q_d = Q_T \Bigg[1 - \frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d} (1 - e^{-\Delta t^n/\tau^n})}{\tau^n e^{-\Delta t^d/\tau^d}(1 - e^{-\Delta t^n/\tau^n}) + \tau^d (1 - e^{-\Delta t^d/\tau^d})}$$

$$- \frac{m^* F U_{SA} (1 - e^{-\Delta t^n/\tau^n}) \tau^n [\tau^d (1 - e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}]}{\Delta t^d ((1-B) U_{SA} + B U_{co,s})(\tau^n e^{-\Delta t^d/\tau^d}(1 - e^{-\Delta t^n/\tau^n}) + \tau^d (1 - e^{-\Delta t^d/\tau^d}))}$$

$$\frac{L_1 \tau^n e^{-\Delta t^d/\tau^d} (1 - e^{-\Delta t^n/\tau^n}) + L_2 \tau^d (1 - e^{-\Delta t^d/\tau^d})}{(\tau^n e^{-\Delta t^d/\tau^d}(1 - e^{-\Delta t^n/\tau^n}) + \tau^d (1 - e^{-\Delta t^d/\tau^d})} \Bigg]$$

J/m^2 day

(1.7.23)

$$
\begin{aligned}
Q_n = Q_T \Bigg[& \frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})} \\
& + \frac{m^* F U_{SA}(1-e^{-\Delta t^n/\tau^n})\tau^n \left[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}\right]}{\Delta t^d((1-B)U_{SA} + B U_{co,s})\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]} \\
& - \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + L_2\tau^d(1-e^{-\Delta t^d/\tau^d})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}
\end{aligned}
$$

$$J/m^2\,day \qquad (1.7.24)$$

and from $SHF_d^{(o)} = C_o^d\, SLR$ we obtain

$$
\begin{aligned}
C_o^d = (\bar{L}/\bar{L}_d)\Bigg[1 - & \frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})} \\
& - \frac{m^* F U_{SA}\tau^n(1-e^{-\Delta t^n/\tau^n})\left[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}\right]}{\Delta t^d((1-B)U_{SA} + B U_{co,s})\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]} \\
& + \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + L_2\tau^d(1-e^{-\Delta t^d/\tau^d})}{Q_T\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]} \Bigg]
\end{aligned}
$$

$$(1.7.25)$$

$$
\begin{aligned}
C_o^n = (\bar{L}/\bar{L}_n)\Bigg[& \frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})} \\
& + \frac{m^* F U_{SA}(1-e^{-\Delta t^n/\tau^n})\tau^n\left[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}\right]}{\Delta t^d((1-B)U_{SA} + B U_{co,s})\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]} \\
& - \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + L_2\tau^d(1-e^{-\Delta t^d/\tau^d})}{Q_T\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]} \Bigg]
\end{aligned}
$$

$$(1.7.26)$$

where

Q_T $[J/m^2]$

$\tau^d = (mc)_s / ((1-B)U_{SA} + B U_{co,s})$ [Hrs] (1.7.27)

$\tau^n = (mc)_s / (1-B)U_{SA}$ [Hrs] (1.7.28)

$(mc)_s$ = thermal mass of storage (9.2×10^5 $J/m^2 K$)

and

$$m^* = [1-((1-B)F + BF_c)]/F$$
$$= [1-F+BF-BF_c]/F \qquad (1.7.29)a$$

$$L_1 = \frac{B \cdot U_{SA} \cdot U_{cos} \cdot \tau^d (1-e^{-\Delta t^d/\tau^d} - \Delta t^d/\tau^d)(T_c - T_{co}^d) \times 3600}{(1-B)U_{SA} + BU_{co,s}}$$

$$L_2 = \frac{B \cdot U_{SA} \cdot U_{cos} \cdot \tau^n (1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})(T_c - T_{co}^d) \times 3600}{(1-B)U_{SA} + BU_{co,s}} \quad [J/m^2]$$

$$U_{SA} = 17, U_{co,s} = 6 \, W/m^2K \qquad (1.7.29)c$$

$$U_{SA}, U_{co,s} \, [J/m^2 K\,hr] = [W/m^2 K] \times 3600.$$

T_c and T_{co}^d are the room and conservatory temperatures during the day respectively. T_{co}^d is given by eqns 2.43., ie.

$$T_{co}^d = \left[\left[\frac{F_R U_{eq} [BF_c Q_T + (1-B)FQ_T]/U_{eq}}{F_R U_{eq} + \dot{m}_{infil} C_p} \right] + T_a \right] \,{}^\circ K$$

$$(1.7.30)a$$

$$T_c = 18.3^\circ C = 291.3^\circ K$$

The equivalent heat transfer coefficient is from eqn. (2.2)

$$U_{eq} = \frac{B(1-B)U_{SA} \; U_{cos}}{(1-B)U_{SA} + BU_{cos})} \qquad (1.7.31)$$

U_{SA} and U_{cos} are the heat transfer coefficients from storage to room air and conservatory to storage. The heat removal factor for the building envelop treated as a collector is from eqn. 2.11.

$$F_R = \frac{\dot{m}_{infil} C_p}{U_{eq}} \left(1 - e^{-[U_{eq}/\dot{m}_{infil} C_p]}\right)$$

$$(1.7.32)$$

in eqns 1.2.30-32.

$\dot{M}_{infil}$ is in ($kg/m^2 s$)(cross section of flow). U_{eq} in (W/m^2K). Q_T the transmitted insolation in ($W/m^2/day$). C_p the specific heat capacity for air at constant pressure, taken as 1012 ($J/kg\,K$) B the ratio representing the size of conservatory compared with the building. Size (cons.) + size (B.) = 1. T_a the ambient temperature (T_a = 11.6 ($284.6^\circ K$). Δt^d in eqns. 1.7.23 - 1.7.29 is taken as a day length of 10 hours (in hours). F and F_c the fraction of transmitted insulation into room and conservatory not absorbed by storage, e.g. F = 0.2, F_c = 0.5 (F << F_c). If the windows and doors in house are assumed closed, then air moves in by infiltration. Hence the use of $\dot{m}_{infil}$ in eqns. 1.7.29 a-d. When open, wind and stack forces come to place, and $\dot{m}_{infil}$ is superseded. In that case $\dot{m}_{vent}$ is used. Derivations for $\dot{m}_{infil}$ and $\dot{m}_{vent}$ [kg/m^2s] of flow (see eqns. (2.35) and (2.66); lead to

$$\dot{m}_I = A_o \times \frac{\rho_{atm}}{R_a T_{a,c}} \left[\frac{\left[\frac{0.2C^* V_w^2}{2T_a} - 0.5 g H \left(\frac{1}{T_a} - \frac{1}{T_c} \right) \right] \frac{P_{atm}}{R_a}}{\frac{\left[\frac{0.2C^*(11.2)^2}{2T_a} - 0.5 g H \left(\frac{1}{T_a} - \frac{1}{T_c} \right) \right] \frac{P_{atm}}{R_a}}{(0.00254)^{1.5}}} \right]^{0.66} \quad (kg/s) \qquad (1.7.33)$$

and

and

$$\dot{m}_{v,st,wind} = \left[\frac{355.5}{\left[100 \left[\frac{\dot{m}_{v,st}}{\dot{m}_{v,st} + \dot{m}_{v,wind}} - 0.1 \right] \right]^{1.9}} + 1 \right] \dot{m}_{v,st}$$

(1.7.34)

where the air flow rate by ventilation (wind and stack) are $\dot{m}_{v,wind}$ & $\dot{m}_{v,st}$ in (Kg / S) respectively and from equations 2.54 a-d and 2.55 a-d.

$$\dot{m}_{v,wind} = \frac{P_{atm}}{R_a T_{a,c}} \left[V_w^2 \times 0.5 \times 10^{-3} A \right] \left[1 + .2415 \left(\frac{A_{out}}{A_{in}} - 1 \right)^{0.37} \right]$$

(Kg/s)

(1.7.35)

$$\dot{m}_{v,st} = \frac{P_{atm}}{R_a T_{a,c}} \times 10^{-3} \times 89 A \sqrt{0.5 H (T_c - T_a)^*} \left[1 + .2415 \left(\frac{A_{out}}{A_i} - 1 \right)^{.37} \right]$$

Kg/s

(1.7.36)

$$A = \begin{cases} A_{out} & (A_{out} < A_i) \\ A_i & (A_{out} > A_i) \end{cases} \quad (1.7.37)$$

$$T_{a,c} = \begin{cases} T_a & (T_a > T_c) \\ T_c & (T_a < T_c) \end{cases} \quad (1.7.37)a$$

Also

$$(T_c - T_a)^* = T_a - T_c \quad (T_c < T_a) \quad (1.7.37)b$$

Finally, P_{atm} is the outside atmospheric pressure, taken as P_{atm} = 101325 Pa and R_a the universal gas constant for air (R_a = 287.045 J/Kg K,air). Of course H and V_w is the height of building e.g. (H = 4m) and wind speed (half season average e.g. 5 m/s).

(B) <u>The case of the NO night insulation</u> (A14)

The average useful daytime (night time) energy gain Q_d^* (Q_n^*) are identical to the case with night insulation with L_2 replaced by (L_2 + L_3). The same is true for the coefficients C_o^{d*} and C_o^{n*} . See discussion (A.12).

$$L_2 = L_2 + L_3 \qquad (1.7.37)c$$

$$Q_d,\ Q_n,\ C_o^d,\ C_o^n \quad (\text{same})$$

where L_3 is given from (A11) as

$$L_3 = -\frac{B \cdot U_{SA}\, U_{cos} \cdot \tau^n (1 - e^{-\Delta t^n/\tau^n} - \Delta t^n/\tau^n)\, K^* (T_c - T_{co}^d)}{(1-B)\, U_{SA} + B\, U_{co,s}}$$

$$(1.7.37)d$$

where

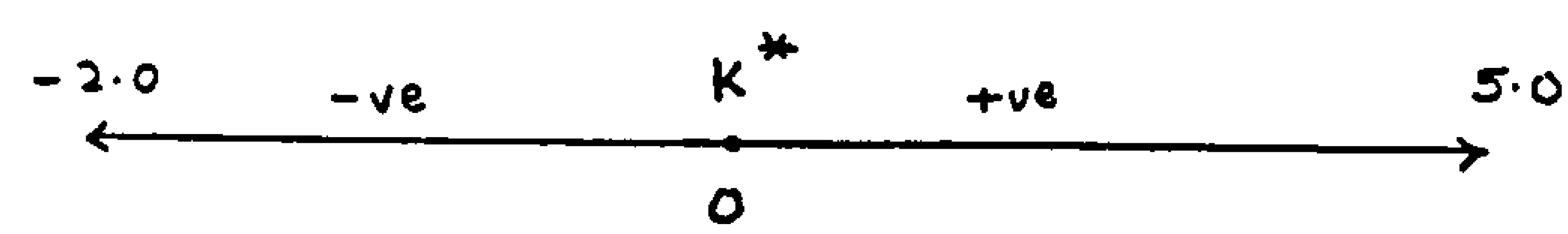
$$\tau = \tau^d = \tau^n = (mc)_s / ((1-B)\, U_{SA} + B U_{co,s})$$
$$(1.7.37)e$$

$$K^* = (T_c - T_{co}^n) / (T_c - T_{co}^d)$$
$$(1.7.37)f$$

A(13) presents, K* values lie in the range

$$-2.0 \leq K^{*} \leq 5 \qquad (1.7.31)_9$$

The fraction K* is fully described below (see A.13).



Colder days and nights in conservatory

warmer days and nights in conserv-atory

The typical K* value for our "reference" house with reference conditions (A13) for a given T_{co}^{d} is

$T_{co}^{d} = 25.1°C$

$T_{co}^{n} = 14°C$

$T_c = 18.3°C\ (65°F)$

$K^* = -0.6$ (See Fig. A(13))

K* is a climatic factor.

We note that for the case where night insulation is employed we may assume since no heat proceeds from room to conservatory $(T_c - T_{co}^{n}) = 0$, and $\tau^d \neq \tau^n$ but are given by 1 7.27 and 1.7.28.

Hence from eqn 1.7.37f, K* = 0 and the expressions for C_o^{d*}, C_o^{n*} become identical to the case with night insulation, i.e. $L_3 = 0$. Therefore a value of K* = 0, represents the case with night insulation, If ($\tau^d \neq \tau^n$), eqns. no insulation become identical to those for night insulation, f (L_2), where τ^d, τ^n are now given by eqn 1.7.27, 28.

With these computed values for $C_o^d, C_o^n, C_o^{d*}, C_o^{n*}$ from equations 1.7.25 - 37g, the use of eqn 1.3.7/8 and then 1.3.6 yield the values for SHF as a function of all the parameters appearing in eqns 1.7.25 - 37g, where in eqns 1.3.7/8

$$SLR_{min,d} = 1 / C_o^d \qquad (1.7.38)$$

and

$$SLR_{min,n} = 1 / C_o^n \qquad (1.7.39)$$

and also

$$SHF = (\bar{L}_d / \bar{L})\, SHF^{day} + (\bar{L}_n / \bar{L})\, SHF^{night} \qquad (1.7.40)$$

1.8 The Effect on Qd, Qn And SHF of the Daytime losses L_1 from room air at T_c to conservatory. $f(T_c - T_{co}{}^d)$. (Case with conservatory and Night Insulation.)

In this section we develop expressions for $Q_d{}^{(1)}$, $Q_n{}^{(1)}$, Q_d & Q_n, and $C_o{}^d$ $C_o{}^n$; in terms of the losses from room air to conservatory at daytime, L_1. This permits us to study how these losses will affect the coefficients $C_o{}^d$ and $C_o{}^n$; hence SHF.

We also justify the solution for $C_o{}^d$ and $C_o{}^n$ in our case, by assuming that the conservatory is re moved (B = 0), hence the losses ($L_1 = 0$, $L_2 = 0$) equal zero. The solution for $C_o{}^d$ and $C_o{}^n$ are shown to reduce to the form presented by Gordon and Zarmi (1), (i.e. for a direct gain system with no attached conservatory).

From A15 we obtain the solution for the case of (night insulation) where L_1, L_2 represents the reduction in Q_d, Q_n (daytime, night time useful solar gains), due to daytime losses L_1.

Also, from equations (1.7.36) & (1.7.37), L_1

$$L_2 = L_1\left[\frac{\tau^n(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^n/\tau^n})}{\tau^d(1-e^{-\Delta t^d/\tau^d}-\Delta t^d/\tau^d)}\right] \quad (1.8.1)$$

We can then study how the daytime losses L_1 affects the coefficients C_o^d, C_o^n and hence SHF. This helps us to see the magnitude of error introduced in the analysis by assuming that these losses are zero, i.e. $L_1 = 0$, from when L_1 varies to a typical value.
From (A15), the daytime useful gain is

$$\therefore Q_d = Q_T\Bigg\{\frac{F\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}$$

$$- \frac{mF\,U_{SA}(1-e^{-\Delta t^n/\tau^n})\,\tau^n\left[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}\right]}{\Delta t^d\left[(1-B)U_{SA}+BU_{cos}\right]\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]}$$

$$+ \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + L_2\tau^d(1-e^{-\Delta t^d/\tau^d})}{Q_T\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]}\Bigg\}$$

(1.8.2)

and since $Q_n = Q_T - Q_d$, for days during which back-up is required ($Q_R = 0$)

$$Q_n = Q_T\Bigg\{1 - \Bigg[\frac{F\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}$$

$$- \frac{mF\,U_{SA}(1-e^{-\Delta t^n/\tau^n})\,\tau^n(\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d})}{\Delta t^d\left[(1-B)U_{SA}+BU_{cos}\right]\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]}$$

$$+ \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + L_2\tau^d(1-e^{-\Delta t^d/\tau^d})}{Q_T\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]}\Bigg]\Bigg\}$$

(1.8.3)

Where

$$m = \eta / F = (1 - F + BF - BF_c)/F \qquad \text{Eqn (1.7.25)}$$

$$(1.8.3)a$$

$$\tau^d = (mC)_s / ((1-B)U_{SA} + BU_{cos}) \qquad \text{Eqn (1.7.4)} \qquad (1.8.3)b$$

and $\tau^n = (mC)_s / (1-B)U_{SA}$ (1.8.3)c Eqn (1.7.10)

We now proceed by evaluating the coefficients C and C by the same procedure detailed in the sections proceeding eqn (1.3.25). Hence from eqn (1.8.2) to (1.8.3), for

(a) Night Time Insulation

$$C_o^d = (\bar{L}/\bar{L}_d)\left[\frac{F\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}\right.$$

$$- \frac{mFU_{SA}(1-e^{-\Delta t^n/\tau^n})\tau^n[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}]}{\Delta t^d((1-B)U_{SA} + BU_{cos})[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})]}$$

$$\left. + \frac{[L_1\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + L_2\tau^d(1-e^{-\Delta t^d/\tau^d})] \times 3600^2}{Q_T[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})]}\right]$$

$$-(1.8.4)$$

m is given by eqn. 1.7.25.

τ^d by eqn. 1.7.4.

τ^n by eqn. 1.7.10.

Hence, from (1.3.22) - (1.3.25). same procedure

$$c_o^n = (\bar{L}/\bar{L}_n)[1 - c_o^d(\bar{L}_d/\bar{L})] \qquad (1.8.5)$$

where

L_1 is given in A9 as (also eqn 1.7.36)

$$L_1 = \frac{U_{SA} U_{cos} B \tau^d (1 - e^{-\Delta t^d/\tau^d} - \Delta t^d/\tau^d (T_c - T_{co}^d)}{[(1-B)U_{SA} + B U_{cos}]} \qquad (1.8.6)$$

and from A10 (see also eqn (1.7.19))

$$L_2 = \frac{U_{SA} U_{cos} B \tau^n (1 - e^{-\Delta t^d/\tau^d})(1 - e^{-\Delta t^n/\tau^n})(T_c - T_{co}^d)}{[(1-B)U_{SA} + B U_{cos}]} \qquad (1.8.7)$$

with m, τ^d & τ^n as previously defined above.

From equation (1.8.6), we can study how the daytime losses from room air at T_c, to conservatory at T_{co}^d, (L_1), vary with the temperature difference between conservatory & room air. ($T_c - T_{co}^d$).

In cases where $T_{co}^d > T_c$ then ($T_c - T_{co}^d$) is -ve. In such cases we have L_1 being -ve, and from eqn (1.7.17), - (-ve L_1), gives the addition to $Q_d^{(1)}$ by conservatory. Therefore, If $T_{co}^{d} > T_c$, L_1 is (-ve), and represents addition to $Q_d(1)$by the attachment of a conservatory. From eqn. (1.8.6) and (1.8.7) we can therefore plot L_1 and L_2 as functions of $(T_c - T_{co}^{d})$, B, e.t.c.

<u>The Case of No Conservatory</u> (B = 0)

We shall now justify our solution for C_o^d and C_o^n by ensuring they reduce to that for a direct gain system (d.g.s) without an attached conservatory for value of B = 0. The results for the d.g.s. was presented by Gordon and Zarmi.(1)

If there is no conservatory attached to our system, then for our case (with night time insulation), the terms L_1 andL_2 which represent losses or gains between the consvatory and room, i.e. f's $(T_c - T_{co}^{d})$, both vanish in the expression for C_o^d. (Eqn. (1.8.4)).

i.e. $L_1 = L_2 = 0$ for B = 0.

For B = 0, Eqn. 1.8.3b & 3c gives $\tau^d = \tau^n = (mC)_s / U_{SA}$. Substituting these values in eqn. 1.8.4 gives; for night time insulation and,

No conservatory.

$$B=0, \quad L_1=0, \quad L_2=0, \quad \tau^d = \tau^n = \tau \tag{1.8.8}$$

$$\xrightarrow{1.8.3a} \quad mF = (1 - F + BF - BF_c)\,F/F, \quad B=0$$

$$mF = (1-F) \tag{1.8.9}$$

$$\xrightarrow{1.8.4} \quad C_o^d = \bar{L}/\bar{L}_d \left[\frac{F\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})}{\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})} \right.$$

$$- \frac{(1-F)(1-e^{-\Delta t^n/\tau^n})\,U_{SA}\,\tau^n\left[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}\right]}{\Delta t^d\,U_{SA}\left[\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \tau^d(1-e^{-\Delta t^d/\tau^d})\right]}$$

$$\left. - \, 0 \right] \tag{1.8.10}$$

we add and subtract $\tau^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n})$
in numerator of term 1. and combine the added part with term 2 of (term 1 of the eqn).

subtracted part

$$\therefore C_o^d = (\bar{L}/\bar{L}_d)\left[1 - \frac{(1-F)\chi^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n})}{\chi^n e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + \chi^d(1-e^{-\Delta t^d/\tau^d})}\right.$$

$$\left. - \frac{(1-F)\tau(1-e^{-\Delta t^n/\tau^n})\left[(1-e^{-\Delta t^d/\tau^d}) - \frac{\Delta t^d}{\tau}e^{-\Delta t^d/\tau^d}\right]}{\Delta t^d\left[e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^n/\tau^n}) + (1-e^{-\Delta t^d/\tau^d})\right]}\right] \tag{1.8.11}$$

$$C_o^d = (\bar{L}/\bar{L}_d)\left[1 - \frac{(1-F)\tau(1-e^{-\Delta t^n/\tau})}{\Delta t^d\left[1-e^{-(\Delta t^d+\Delta t^n)/\tau}\right]}\left[\cancel{\frac{\Delta t^d}{\tau}e^{-\Delta t^d/\tau^d}} + (1-e^{-\frac{\Delta t^d}{\tau}}) - \cancel{\frac{\Delta t^d}{\tau}e^{-\frac{\Delta t^d}{\tau}}}\right]\right] \tag{1.8.12}$$

$$C_o^d = (\bar{L}/\bar{L}_d)\left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^n/\tau})(1-e^{-\Delta t^d/\tau})}{\Delta t^d\left[1-e^{-(\Delta t^d+\Delta t^n)/\tau}\right]}\right]\right] \tag{1.8.13}$$

Thus for the case with no conservatory, B = 0, we see that our general expression reduces to that by Gordon and Zarmi (1).

Hence we use the relation between L_2 and L_1, eqn. 1.8.1 and substituting $L_2 = f(L_1)$ in eqn 1.8.4 gives C_o^d as a function of L_1. From the eqns for SHF^{day} and SHF^{night} i.e, 1.3.7, 1.3.8 and eqn 1.3.6 for SHF, we obtain SHF = $f(L_1)$ where L_1 is the gain from conservatory to room air during daytime.

We also note that in eqn. (1.3.6)

$$(\bar{L}_d/\bar{L}) + (\bar{L}_n/\bar{L}) = 1 \tag{1.8.14}$$

and in eqns. (1.3.7) and (1.3.8)

$$SLR_{min,d} = 1 / C_o^d \qquad (1.8.15)$$

$$SLR_{min,n} = 1 / C_o^n \qquad (1.8.16)$$

δ in eqn. (1.3.7) and (1.3.8) was assumed a value of 0.8.

Thus we see that our analysis for a direct gain system with conservatory reduces to the case of a direct gain system, (for B = 0), validating our results. This is for the system with night insulation. We can now see how the losses from room air to conservatory at day L_1 affects SHF by plotting

$$SHF = f(L_1) \qquad (1.8.17)$$

Also, for different sizes of conservatory we can study how the SHF varies, i.e.

$$SHF = f(B) \quad , \quad 0 \leq B \leq 1.0 \qquad (1.8.18)$$

Eqn. 1.8.18 represents buildings of different geometric designs, from no conservatory, B = 0, the direct gain (1); to the very large conservatory (B = 1.0) greenhouse.

1.9. Sample Calculation

(i) Direct gain, Conservatory, night insulation

We specify the "reference" house considered by Gordon and Zarmi (1), from ref.(3) with the following properties.

$\bar{L}/\bar{L}_d = 3 \qquad \bar{L}/\bar{L}_n = 3/2$

$U_{SA} = 17\ W/m^2K = 17 \times 3600\ J/m^2K\,hr$

$(mC)_s = 9{\cdot}2 \times 10^5\ J/m^2K$

$\Delta t^d = 10$ hours

$\Delta t^n = 14$ hours

$Q_T = 1{\cdot}224 \times 10^9\ J/m^2 \equiv 8500\ W/m^2$

$F = 0{\cdot}36 \qquad \bar{T}_c[1] = 18{\cdot}3^\circ C$

$T_a = 11{\cdot}6^\circ C \qquad 1-F = 0{\cdot}64$

$T_a^d = 14{\cdot}5^\circ C$

$\bar{T}_{co}^d = 25{\cdot}1^\circ C$ (assumed)

$\bar{T}_{co}^n = 14^\circ C$ (assumed for heating season)

$F_c = 0{\cdot}5$ (assumed)

$B = 0{\cdot}2$ (assumed)

$U_{cas} = 1/3\ U_{SA}$ (assumed Low) $= 6{\cdot}0\ W/m^2K \cdot (6 \times 3600\ J/m^2K\,hr)$

$k^* = (T_c - T_{co}^n)/(T_c - T_{co}^d) = -0{\cdot}6$

Eqn. 1.7.27

$\tau^d = (mC)_s / [(1-B)U_{SA} + BU_{co,t}]$

$\tau^d = 9.2 \times 10^5 / [0.8(17 \times 3600) + 0.2(6 \times 3600)]$

$\tau^d = 17$ hrs (Computed)

Eqn. 1.7.28

$\tau^n = (mC)_s / (1-B)U_{SA}$

$\tau^n = 18.8$ hrs (Computed)

$m = (1-F+BF-BF_c)/F = 0.61/0.36$

$m = 1.7$

$mF = 0.61$

The mass surface absorptance of the wall $S_A = 0.8$. Strictly speaking mF should be

$mF = S_A(1-F+BF-BF_c)$

$\overset{1.8.4}{\Longrightarrow}$ (for night insulation)

C_o^d = Known (1.9.1)

$C_o^n = (\bar{L}/\bar{L}_n)[1-C_o^d(\bar{L}_d/\bar{L})]$ (1.9.2)

In the above eqns the dimension (L_1) is (J/m^2) and (Q_T) is (J/m^2) per day to be used. (τ) hours. e.g. Q_T = 1.224×10^{-9} J/m^2 / 120 = 1.02×10^{-7} J/m^2 day.

With the above information the following parameters are caclulated as

$$L_1 = 24.3\ J/m^2$$
$$L_2 = -41.9\ J/m^2$$
$$L_3 = 24.7\ J/m^2$$
$$(T_c - T_{co}^d) = -6.8\ ^\circ C$$
$$\frac{3600^2}{Q_T} = 0.01$$
$$C_o^d = 0.84$$
$$C_o^n = 1.08$$

Hence for $\overline{SLR}$ = 1.0, δ = 0.8

SHF^d = 0.77

SHF^n = 0.88

and SHF = 0.85 for $\overline{SLR} = 1.0$ d.g conservatory night insulation] our model + ref (1)

d.g SHF = 0.7 for $\overline{SLR} = 1.0$ night insulation Fig 2.(1)

Thus we see the introduction of the conservatory on our reference house (1), increase the SHF by 0.14 (20%).

(ii) <u>Direct Gain, Conservatory, no night insulation</u>

Here we simply note eqn 1.9.1 is same with L_2 replaced by ($L_2 + L_3$) and $\tau^d = \tau^n =$

$(mC)_s / ((1-B) U_{SA} + B U_{co,s})$ (17 hours)

$\overset{1.9.1}{\Longrightarrow}$ $C_o^d = 1.31$ $\qquad$ $\overset{1.9.2}{\Longrightarrow}$ $C_o^n = 0.84$

$SHF^d = 0.93$ $\qquad$ $SHF^n = 0.78$

$\therefore$ SHF = 0.83 for $\overline{SLR} = 1.0$

Comparing cases (i) night insulation and (ii)no insulation we see in the later SHF^d increases, SHF^n decreases and SHF is lower than in (i) with night insulation.

Thus the contribution by using night insulation is 2.4%;(.85 - .83)/.85;increase in SHF compared to no night insulation. Thus night insulation is insignificent at SLR = 1.0, for a d.g house with attached solarium.

A few useful keynote deductions thus far are

1. An attached conservatory 1/4 the size of the d.g room B = 0.2 (20%), increases SHF by 20%.

   This is in comparison to the "reference" house by Gordon and Zarmi(1).

2. Night insulation for a d.g. house with attached atrium increases SHF by 2.4% compared to no night insulation.

3. The model presented in this report reduces to the Direct Gain house without conservatory for B = 0

   i.e. $\tau^d = \tau^n = (mC)_s / U_{SA}$

4. We note we assumed a rate of heat transfer from conservatory to storage $U_{co,s}$ of $6 W/m^2 K = 1/3\, U_{SA}$, the heat transfer coefficient from storage to room air.

(iii) No conservatory B = 0 i.e. Direct Gain only

$$f'_s (T_c - T^d_{co}) = 0 \; ; \; L_1 = 0 \, , \, L_2 = 0$$

Night insulation

$$\tau^d = (mC)_s / ((1-B) U_{SA} + B U_{co,s})$$

$$\tau^n = (mC)_s / (1-B) U_{SA}$$

$\therefore\ \tau^d = \tau^n = (mC)_s / U_{SA}$

same as in ref (1), for direct gain house

In section (1.8) we show that our expressions for C , C reduces to the form presented in ref.(1), namely

$$C_o^d = \bar{L}/\bar{L}_d \left[1 - \left[\frac{(1-F)\tau(1-e^{-\Delta t^n/\tau})(1-e^{-\Delta t^d/\tau})}{\Delta t^d \left[1 - e^{-(\Delta t^d + \Delta t^n)/\tau}\right]} \right] \right]$$

see eqns (1.8.13), and proceeding derivations.

Hence as in (1) we obtain

SHF = 0·7

Hence for no conservatory

$B = 0$ \& $\tau^d = \tau^n = \tau = (mC)_s / U_{SA}$

(iv) Factors, to verify influence on d.g. rooms SHF, with conservatory and night insulation.

(a) $\delta/\overline{SLR}$ (e) $\bar{L}/\bar{L}_d$ (i) Δt^d

(b) L_1 (f) $B, B=0$ (j) U_{SA}

(c) L_2 (g) F_c (K) $U_{co,s}$

(d) L_3 (h) F (l) $\tau^d \neq \tau^n$ (night insulation)

<u>No night insulation</u>

Shown insignificant, but we study how outdoor night time conservatory conditions affects SHF. $f(K^*, T_{co}^d, T_{co}^n)$

$-2{\cdot}0 \leq K^* \leq 5{\cdot}0$ (A13)

2.0 Evaluating the Conservatory Temperature

For purposes of evaluating the conservaatory temperature, it is considered that the direct gain system with attached conservatory constitutes a solar energy collecting system, or solar energy collector (SEC). We consider the energy balance on fluid element (air), in the flow direction; in this case assumed along the height of the building as shown in Fig 2.1 •

It is assumed that air inlets are placed at a level of between zero and one metre above floor level in the conservatory and corresponding air outlets are located in the top floor ceilings. The roof void may be ventilated by using a combination of roof ridge and roof tile ventilators.

Assume total area opening of vent for conservatory A_{co} = 5×10^{-4} m^2, and for the ceiling vent A_c = 5×10^{-4} m^2.

The configuration shown in fig 2.1 maximises the roof void suction relative to the conservatory for both wind and stack effects. Furthermore, the direction of air flow is invariant to wind direction and in essence flows the desired flow route (16). The low level of conservatory vents enhances stack flow and eliminates the possibility of stack induced (temp.difference induced) return flow, resulting from conservatory air temperature, T_{co} , being at a higher temperature than the living space. T_c.

The dangers of condensation in the roof space by entering moisture and poor ventilation in the remote zones from the immediate vicinity of the conservatory are eliminated by placing the moisture producing areas, i.e. kitchen and bathroom, in the north facing side of the building and venting them directly to the outside via ventilation stacks. Some ceiling porosity is still required but internal moisture migration is much reduced while flow paths to the remote rooms are also established. This is shown in Figure 2.2.

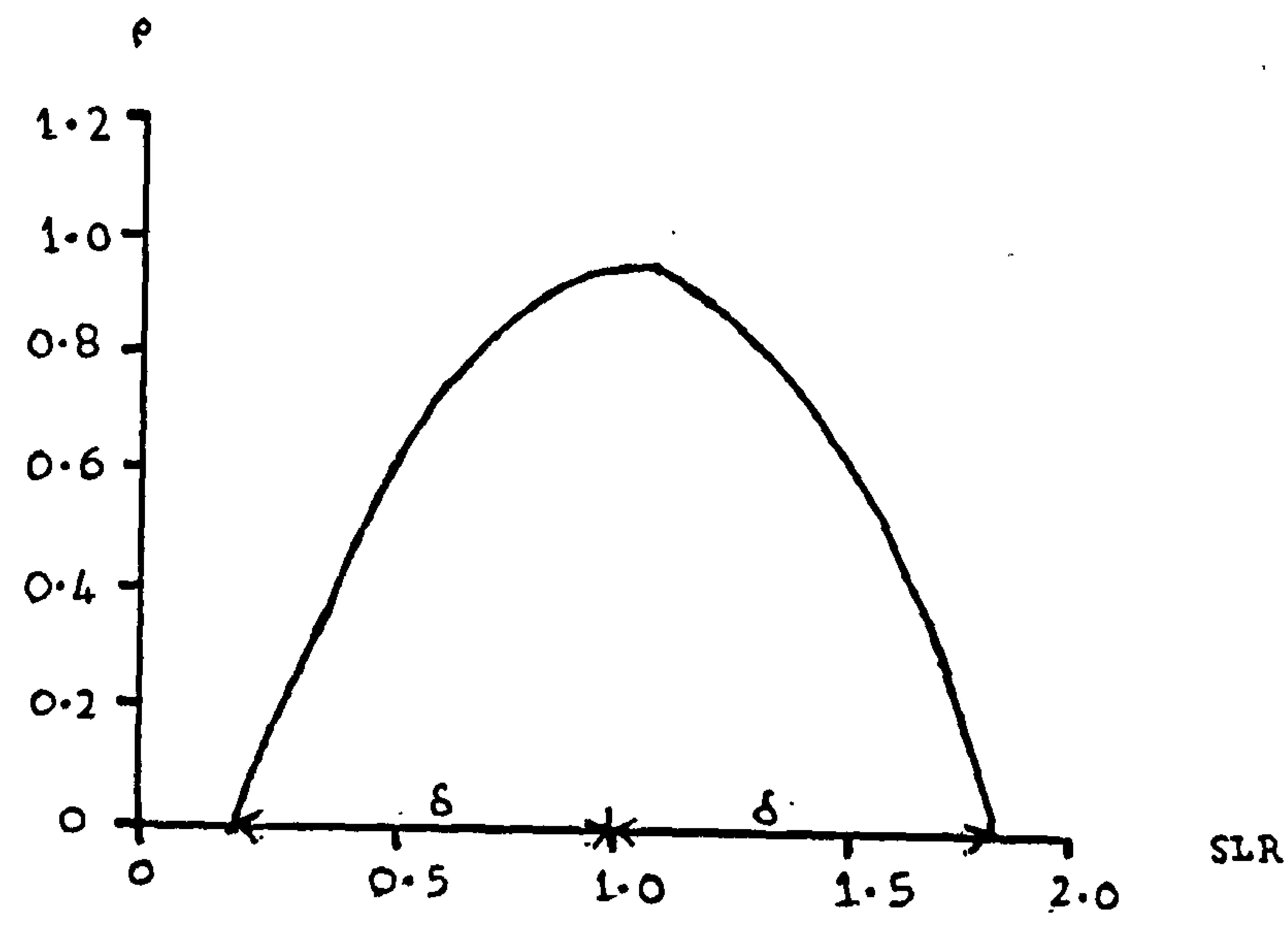


Fig.1

The Parabolic Distribution Function p(SLR)

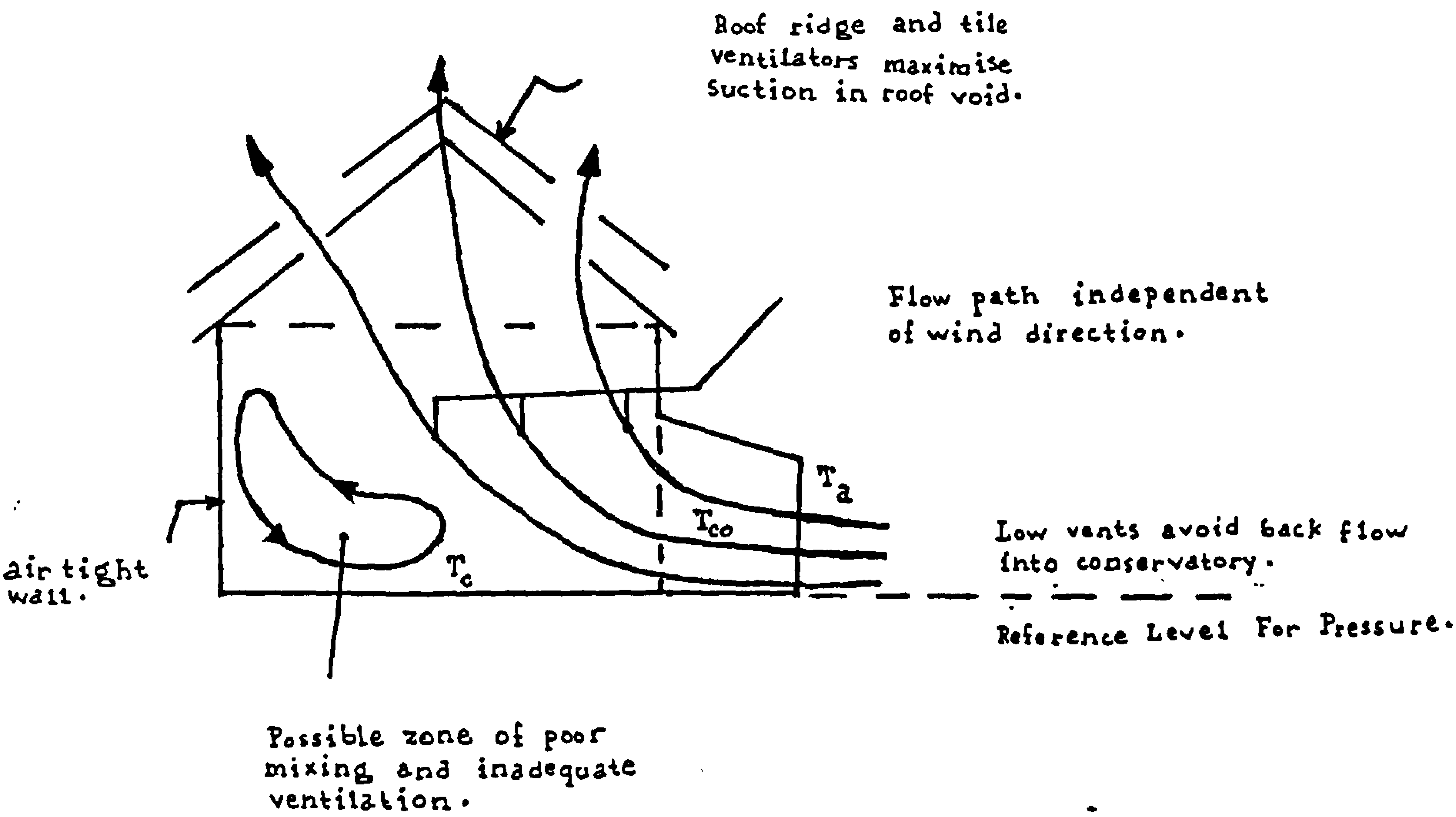


Fig 2.1 Practical Aspects of Designing the Direct Gain System with attached Conservatory

Fig 2.2 is designing for natural ventilation: stack effects.
For the conservatory we present a thermal circuit representing the inlet flow of air from the building envelope assumed at conservatory temperature for purposes of calculating the heat removal factor of the building, (see Figure 2.3). The heat removal factor F_R is the ratio of the useful gain of the building envelope assumed at temperature T_{co}, to its actual value.

From Figure 2.3, the equivalent resistance of the building envelop can be written as

$$1/R_{eq} = 1/R_{SA} + 1/R_{co,a} \qquad (2.2.1)$$

where R_{sa} and $R_{co,a}$ represent the resistance from room air and conservatory to ambient respectively.

Hence,

$$R_{eq} = (R_{sa} R_{co,a}) / (R_{sa} + R_{co,a}) \qquad (2.2.2)$$

and

$$U_{eq} = 1/R_{eq} = (R_{sa} + R_{co,a}) / R_{sa} R_{co,a} \quad (2.2.3)$$

then,

$$U_{eq} = \frac{1/(1-B)U_{sa} + 1/BU_{co,a}}{1/(1-B)U_{sa} \cdot 1/BU_{co,a}} \qquad (2.2.4)$$

and

$$U_{eq} = BU_{co,a} + (1-B)U_{sa} \quad (2.2.5)$$

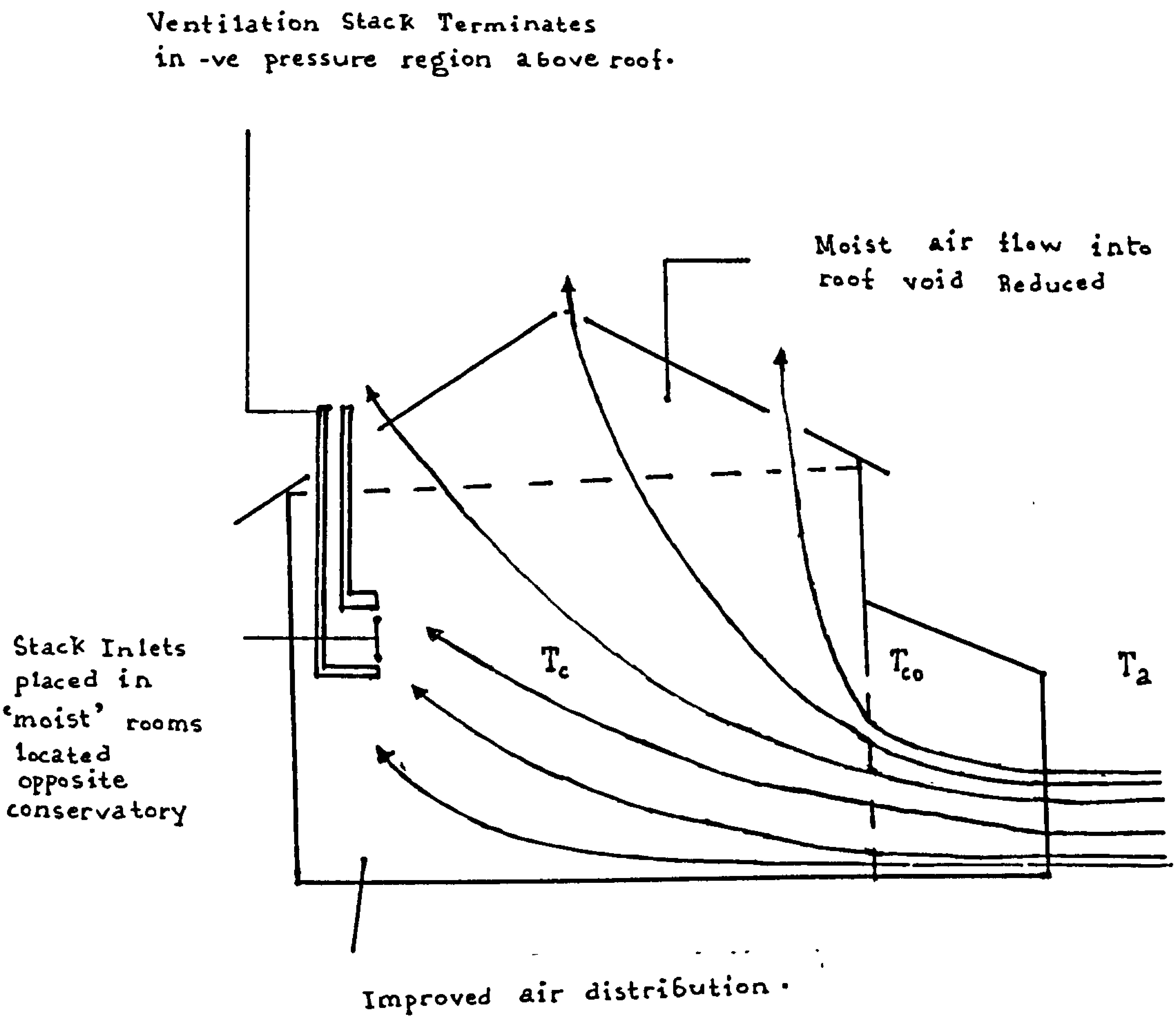


Fig 2.2 Designing for Natural Ventilation:- Stack

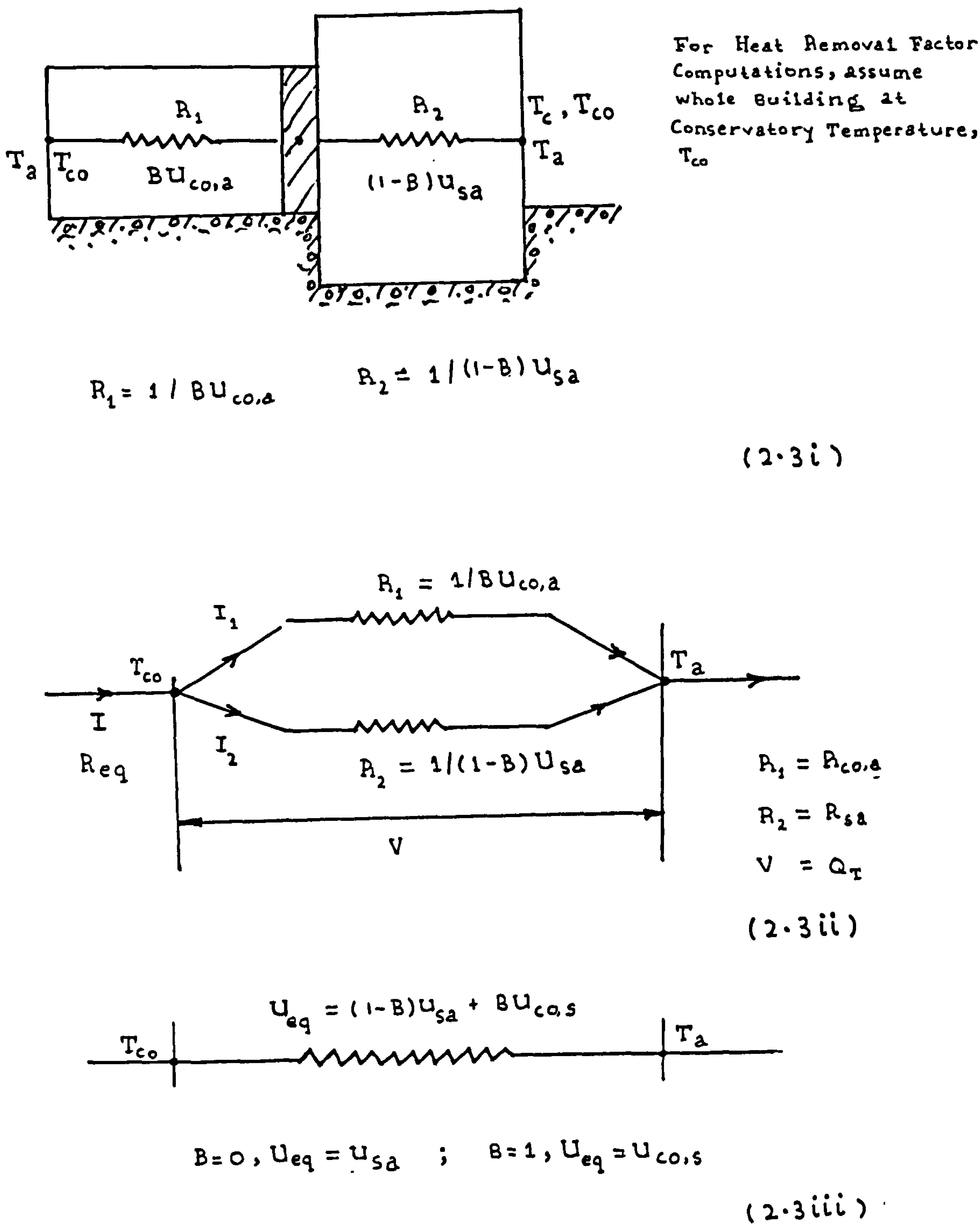


Fig 2.3 The Thermal Circuit of a Direct Gain System with attached Conservatory

We note that in eqn. 2.2.5 when (B = 0) no conservatory, $U_{eq} = U_{sa}$, the heat transfer coefficient from room air to ambient (direct gain); and when B = 1 , the heat transfer coefficient from conservatory to ambient (the greenhouse), $U_{eq} = U_{co,a}$

The useful gain in the thermal circuit of figure 2.3 can be written as

$$q_u = S_1 + S_2 - U_{eq}(T_f - T_a) \qquad (2.3)$$

This is the useful gain assuming the building envelope at fluid inlet temperature T_{co} . We define the heat removal factor shortly which accounts for actual fluctuations in temperature as air proceeds through the building envelope

From eqn. (2.3) above

$$q_u = (BF_cQ_T + (1-B)FQ_T) - U_{eq}(T_f - T_a) \qquad (2.4)$$

where T_f is the conveying air temperature in the direct gain system with conservatory.

The usesful gain given by eqn. (2.4) is ultimately transferred to the fluid. The fluid enters the direct gain system (collector with attached conservatory), at temperature $T_{f,i} = T_{co}$. See Figure 2.4.

Figure 2.4 is schematic design of the direct gain system with conservatory.

Referring to figure 2.5 we can express an energy balance on the fluid flowing through a section of length Δl as

$$\dot{m}C_p T_f - \dot{m}C_p T_f\Big|_{l+\Delta l} + \Delta l\, q_u = 0 \quad (2.5)$$

Dividing through by Δl, the limit as Δl approaches zero, and substituting eqn (2.4) for q_u, we thus obtain

$$\dot{m}C_p \frac{dT_f}{dl} - \left[(BF_c Q_T + (1-B)FQ_T) - U_{eq}(T_f - T_a) \right] = 0 \quad (2.6)$$

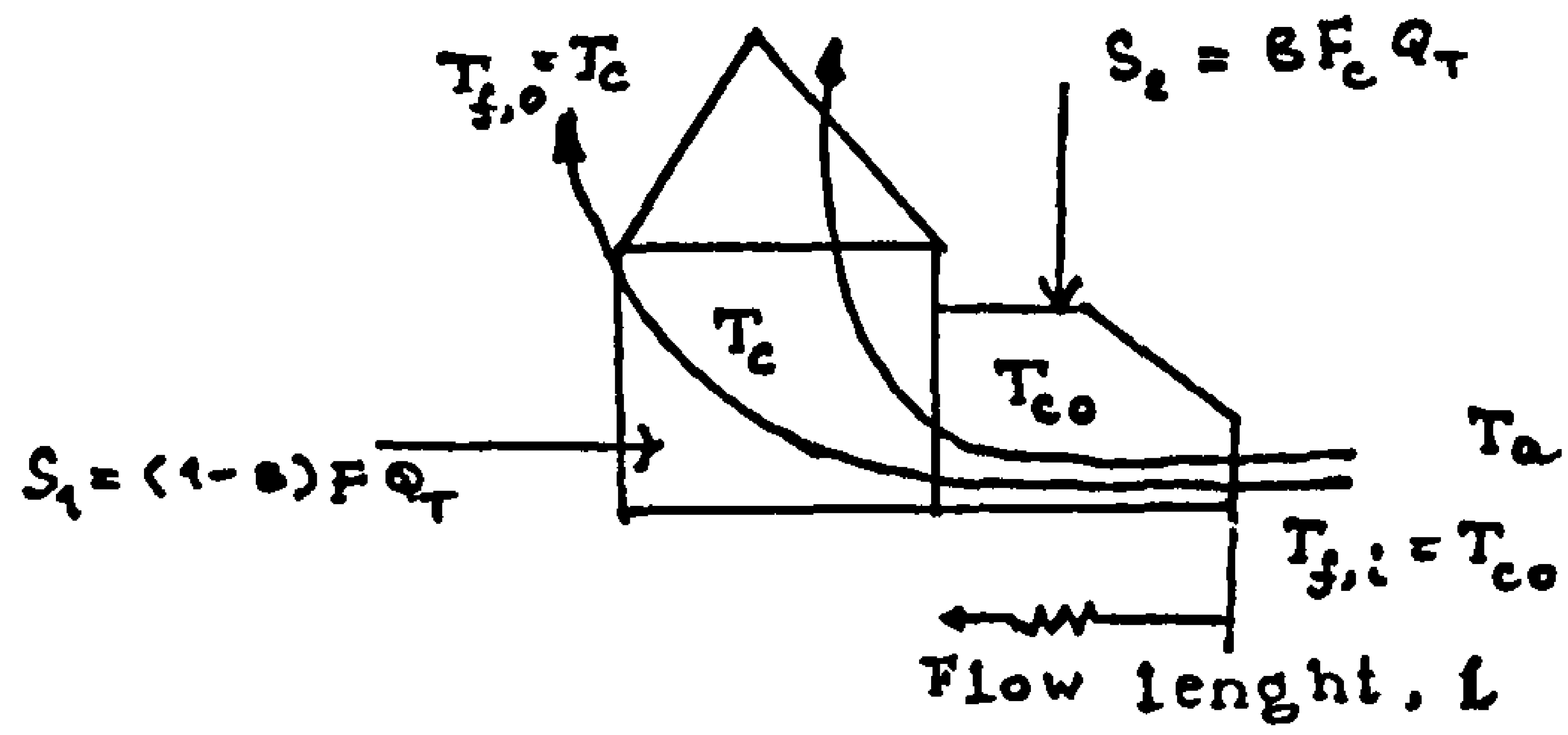


Figure 2.4 Schematic Design of the Direct Gain System with conservatory

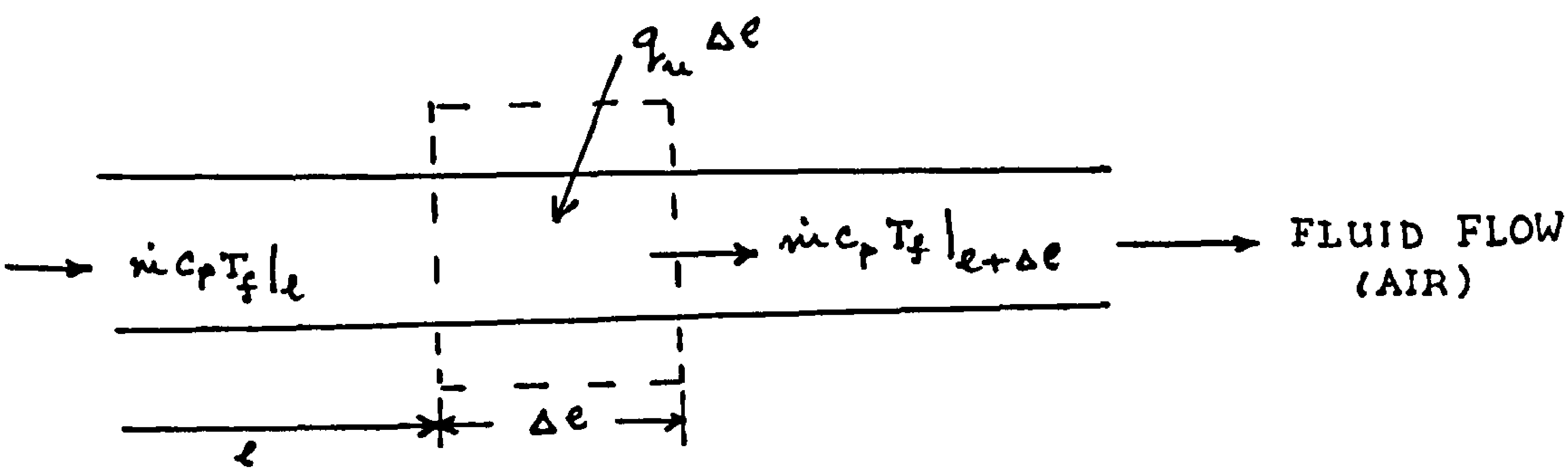


Fig 2.5

Energy Balance on Fluid Element

If we consider that the equivalent loss coefficient U_{eq} for the direct gain system plus conservatory is independent of position, l, then the solution for the temperature at any position l (subject to the condition that the inlet fluid temperature is $T_{f,i} = T_{co}$) is, (see section A.18) for solution.

$$\frac{T_f - T_a - (BF_c Q_T + (1-B)FQ_T)/U_{eq}}{T_{co} - T_a - (BF_c Q_T + (1-B)FQ_T)/U_{eq}} = e^{-[U_{eq} l / \dot{m} C_p]} \tag{2.7}$$

If our system has a length L in the flow direction, then the outlet fluid temperature $T_{f,o} = T_c$, is found by substituting L for l in eqn. (2.7) above.

2.1 Direct Gain System (with attached Conservatory), Heat Removal Factor, F_R and Flow Factor F"

It is convenient to define a quantity that relates the actual useful gain of our system to the useful gain if the whole system were at the fluid inlet temperature, $T_{f,i} = T_{co}$. Mathematically, the collector heat removal factor, F_R, is then

$$F_R = \frac{\dot{m} C_p \cdot (T_{co} - T_a)}{[(BF_c Q_T + (1-B)FQ_T - U_{eq}(T_{co} - T_a)]} \tag{2.8}$$

i.e. what conservatory gives to ambient at exit is the solar input it receives at inlet minus losses (from building envelop to ambient) times the F_R

where $\dot{m}$ is the mass flow rate/metre square of cross section area.
From eqn 2.8 the collector heat removal factor is expressed as

$$= \frac{\dot{m} C_p}{U_{eq}} \left[\frac{T_{co} - T_a}{(BF_c Q_T + (1-B)FQ_T)/U_{eq} - (T_{co} - T_a)} \right] \tag{2.9}$$

and from equation (2.7) when t = L; i.e. the considered flow is at exit , and; for building envelop that flow ceases to receive solar gain; we can then express F_R as

$$F_R = \frac{\dot{m}C_p}{U_{eq}}\left[1 - e^{-(U_{eq}L/\dot{m}C_p)}\right] \qquad (2.10)$$

or with $\dot{m}$ in kg/s of flow cross section area we obtain,

$$F_R = \frac{\dot{m}C_p}{U_{eq}}\left[1 - e^{-(U_{eq}/\dot{m}C_p)}\right] \qquad (2.11)$$

From eqn (2.11) the systems heat removal factor is a function of the single variable $U_{eq}/\dot{m}C_p$.

In eqn. (2.11), $\dot{m}$ is the flow rate in the direct gain system by either infiltration or ventilation (stack and wind); in the flow direction per unit flow area (cross-section) i.e.

$$\dot{m} = \begin{cases} \dot{m}_{infil} & \text{(Infiltration)} \\ \dot{m}_{vent} & \text{(Ventilation)} \end{cases} \qquad (2.12)$$

and $\dot{m}_{infil}$; $\dot{m}_{vent}$ are measured in kg/sm^2 .

As the systems heat removal factor is a function of the single variable$(U_{eq}/\dot{m}C_p)$;a graph can be plotted of F_R Vs $U_{eq}/\dot{m}C_p$.
To fully appreciate the significance of the curve, it is convenient to re-examine eqn. (2.8), but in a slightly different form.

$$Q_u = F_R\left[(BF_cQ_T + (1-B)FQ_T) - U_{eq}(T_{eo} - T_a)\right]$$

where Q_u is the total useful energy gain of our system. With this equation the useful gain is calculated as a function of the inlet fluid temperature. This is a convenient representation when evaluating solar energy systems since the inlet fluid temperature is usually known. However, it must be remembered that losses based on the inlet fluid temperature are too small since losses occur all along the system and the fluid has en ever-increasing temperature in the flow direction. The effect of the multiplier, F_R, is to reduce the calculated useful energy gain from what it would have been had the whole system been at ($T_{f,i} = T_{co}$); to what it actually is using a fluid that increases in temperature as it flows through the system. As the mass flow rate through the system increases, the temperature rise through the system decreases. This causes lower losses and a corresponding increase in the useful energy gain, since the average system temperature is lower.

This increase in useful energy gain is reflected by an increase in the systems heat removal factor as the mass flow rate increases. See eqn. (2.11).

From eqn. (2.13) above

$$\dot{m}c_p(T_{co}-T_a) = F_R\left[(BF_cQ_T + (1-B)FQ_T) - U_{eq}(T_{co}-T_a)\right] \qquad -(2.14)$$

or

$$T_{co}-T_a = \frac{F_R U_{eq}}{\dot{m}c_p}\left[(BF_cQ_T + (1-B)FQ_T)/U_{eq} - (T_{co}-T_a)\right] \qquad -(2.15)$$

$$T_{co}\left[1 + \frac{F_R U_{eq}}{\dot{m}c_p}\right] = \frac{F_R U_{eq}}{\dot{m}c_p}\left[(BF_cQ_T + (1-B)FQ_T)/U_{eq}\right] + T_a\left[1 + \frac{F_R U_{eq}}{\dot{m}c_p}\right] \qquad -(2.16)$$

From which one obtains the conservatory temperature T_{co} in $[°K]$ as

$$T_{co} = \left[\frac{F_R U_{eq}}{\dot{m}C_p + F_R U_{eq}}\left[(BF_c Q_T + (1-B)FQ_T)/U_{eq}\right] + T_a\right] \quad -(2.17)$$

With $T_a = 11.6°C$ (284.6K), and selected parameter values, a study is now possible on T_{co}; namely, how it is affected by such variables as the mass infiltration or ventilation rate per unit area $\dot{m}$; heat removal factor, F_R, given by eqn. 2.10; ambient temperature, and B values i.e

$$T_{co} = f(F_R, U\text{-value}, B, F_c, Q_T, T_a, \dot{m}C_p) \quad -(2.18)$$

$\dot{m}$, in eqn. 2.17 is also a function of the conservatory temperature T_{co}. Therefore, in determining T_{co}, an interactive technique is employed with convergence achieved when $\Delta T = T_{co}^{new} - T_{co}^{old} = 0.1$.

Example

Assume the following values

$B = 0.2$

$U_{sa} = 2.84\ W/m^2K$ (2)

$U_{coa} = 3.0\ W/m^2K$

$F = 0.20$ 20% (2)

$F_c = 0.50$ 50%

$\dot{m}_{infil} = 9.5 \times 10^{-4}\ kg/sm^2$ (infiltration rate into conservatory).

Crack "resistance"

$R_{ci} = 1.3 \times 10^5\ m^3/s\ m^2$ of crack.

T = length of heating season = 120 days (2)

$Q_T = 1.224 \times 10^9\ J/m^2$ (2)

Evaluate the heat removal factor for the above direct gain system with attached conservatory (B = 0.2), a quarter the size of the room; using eqn 2.11, assuming C_p for air is 1012 J/KgK and ambient temperature T_a of T_a = 11.6°C (284.6K) (2)

Solution

Eqn. 2.11 gives the heat removal factor for the above example.

$$F_R = \frac{\dot{m} C_p}{U_{eq}} \left[1 - e^{-(U_{eq}/\dot{m} C_p)} \right] \quad -(i)$$

Using eqn. 2.2.5

$$U_{eq} = (1-B) U_{sa} + B U_{coa} = 2.872 \text{ W/m}^2\text{K} \quad -(ii)$$

(i) & (ii)

$$\Rightarrow \quad F_R = 0.3179 \quad -(iii)$$

Now, evaluate the temperature of the conservatory for the above given conditions.

Using eqn. (2.17) T_{co} is in degrees Kelvin given by,

$$T'_{co} = 0.044\, Q_T + 284.6 \quad {}^\circ K$$

Evaluate Q_T in W/m^2 day using Q_T (120 days)

$$Q_T = 1.224 \times 10^9 \ J/m^2 \ (120 \ days) \quad (2)$$

$$= \frac{1224 \times 10^6}{120 \times 24 \times 3600} \ W/m^2 \ Day.$$

$$= 118.0 \ W/m^2 \ Day.$$

$$\therefore \ T_{co} = 0.044(118.0) + 284.6 \ ^\circ K$$

or

$$T_{co} = 289.8^\circ K \ (16.8^\circ c) \ \text{for} \ T_a = 11.6^\circ C$$

This is the average conservatory temperature during the heating season (120 days), using data in ref. (1,2) and the above analyses.

2.2 The effects of Infiltration, Exfiltration and Natural Ventilation on Conservatory Temperature. T_{co}

In sections 2.0 and 2.1, equation (2.17) presents us with the solution for the conservatory temperature as

$$T_{co} = \left[\left[\frac{F_R U_{eq}\left[(BF_e Q_T + (1-B)F Q_T)/U_{eq}\right.}{F_R U_{eq} + \dot{m} C_p}\right] + T_a\right] \quad -(2.18)$$

$\dot{m}$, in equation (2.18) above represents the air leakage through cracks and interstices, round windows and doors (glazing), into the building (conservatory); i.e. infiltration.

In cases where the occupants of a building desire additional comfort, then they may achieve this by either opening windows and/or doors. Air then proceeds into the building envelop by natural ventilation.

We simplify our analysis by assuming that for natural ventilation, then the cracks and interstices are eliminated. Air movement is now predominantly by natural ventilation from ambient into the building envelop. Natural ventilation is the intentional displacement of air through specified openings such as windows and doors. Section 2.2.1 will analytically evaluate the infiltration or air leakage rate ($\dot{m}$) (kg/s) into the building. In section 2.2.2 we analytically evaluate the air leakage rate into the building when natural ventilation is employed. Then the conservatory temperature's dependence on infiltration or natural ventilation can be studied.

2.2.1 Infiltration Air Leakage Rate into Building

Air flow rate into and out of a building due to either infiltration, exfiltration, or natural ventilation depends on the pressure difference between outside and inside; and on the resistance to flow of air through openings in the building. In this analysis we consider the configuration between ambient at T_a and room air T_e as shown in Figure 2.6.

The amount of air flowing through a crack in an outside wall, window or door, depends on the size of the crack, and the pressure difference between inside and outside.

In most cases, the resistance to air flow within the house is negligible. For the purpose of infiltration calculations, the house can then be regarded as a box, as shown in Figure 2.6.

For all types of windows and sliding glass doors, the Federal Mobile Home Construction and Safety Standard (Fed.MHC SS 280.405), see ASHRAE handbook of fundmentals (17); presents the infiltration through cracks as

$$\rho \dot{V} = \rho c (\Delta P)^{n} \qquad -(2.19)$$

or

$$\rho \dot{V} = \rho c (P_o - P_i)^{n} \qquad -(2.20)$$

In eqn. 2.20, $\rho\dot{V}$, is the weight flow from 0 to i ($kg/s\,m^2$ of A_o); A_o is the air leakage area (m^2). P_o, static pressure at O (see Fig 2.6); n the flow exponent (usually n = 0.66 (18)), and C a flow coefficient.

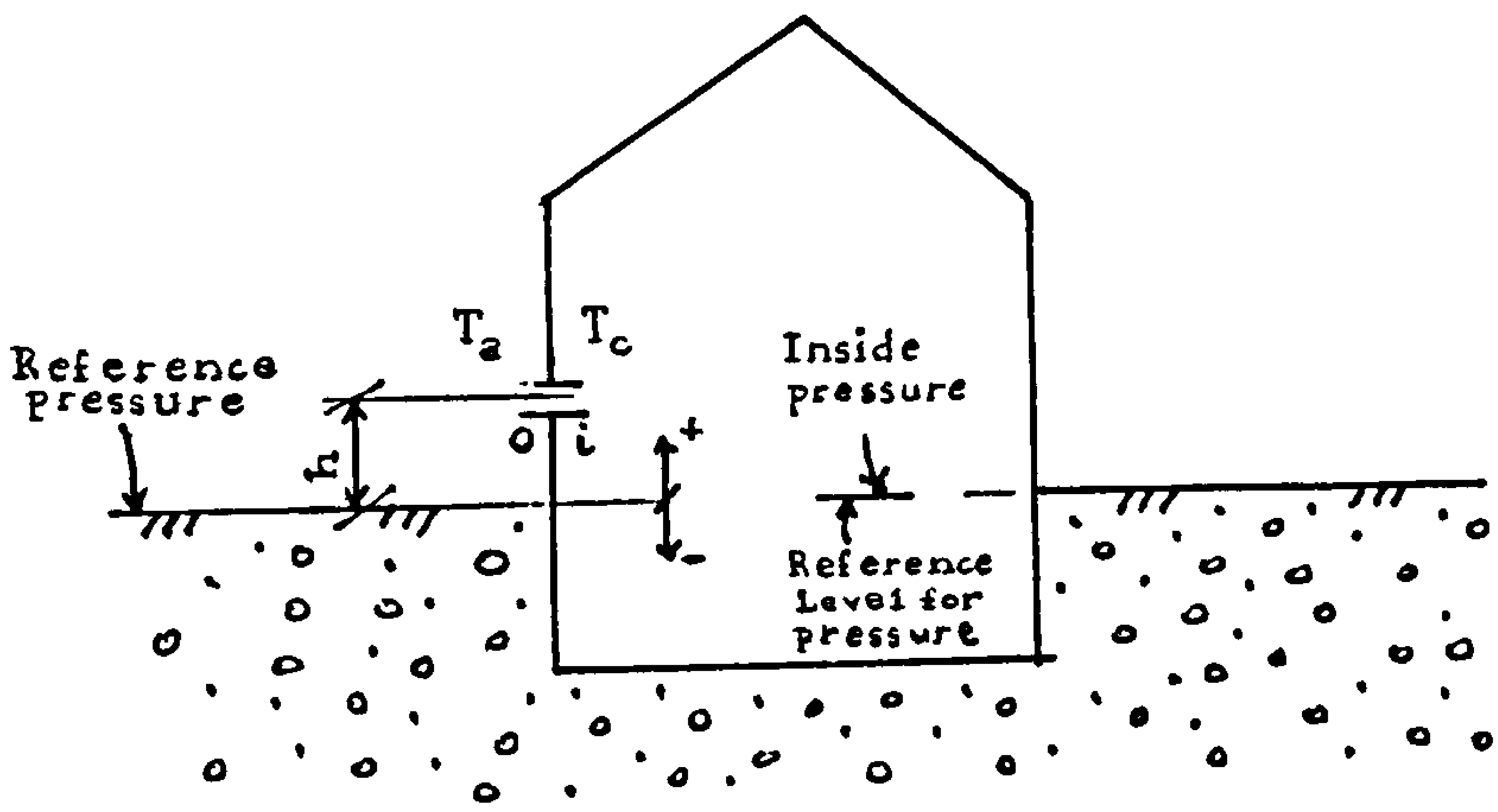


Fig 2.6 Illustration of Infiltration into building from ambient

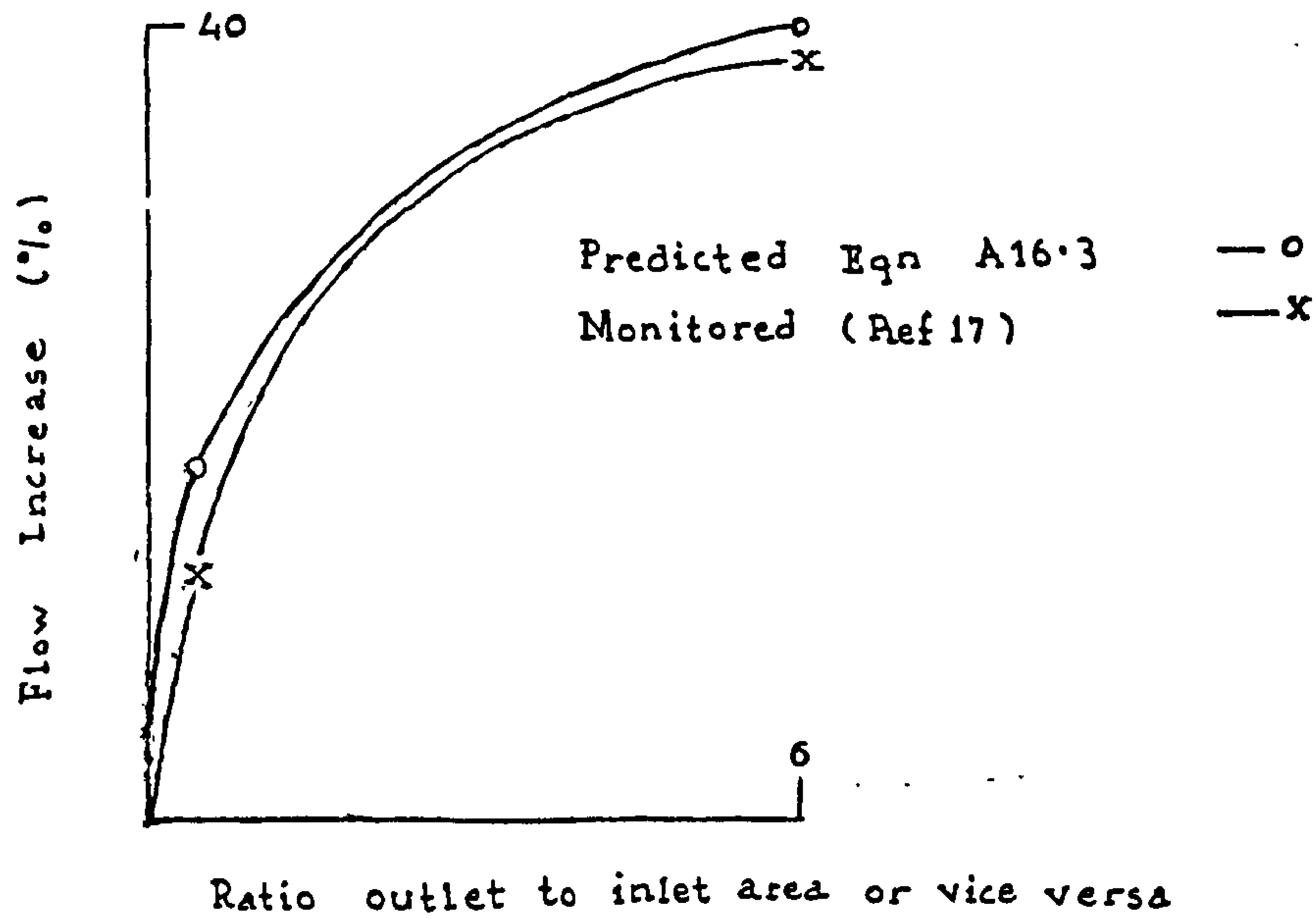


Fig 2.7 Plot, Predicted vs Monitored Air Flow rates increase, resulting from unequal entrance and exit areas.
Eqn. A16.3 (Predicted)
(Ref.17) (Monitored)

Figure 2.8 Diagram for Calculating solar heating fraction for a Direct Gain System with an attached solarium (Conservatory)

Equation (2.20) can be rewritten as

$$\rho \dot{V} = \rho \left[\frac{P_o - P_i}{R_{o,i}} \right]^n \qquad -(2.21)$$

where $R_{o,i}$ is the resistance of the crack, defined as the volume flow rate per unit area of crack, at a unit pressure difference. ASHRAE handbook of fundmentals (17), gives for all types of windows and sliding glass doors

$$\dot{V} = 0.00254 \ m^3/s\,m^2 \text{ of } A_o \qquad -(2.22)$$

at pressure 75 Pascals (N/m^2), equivalent to 11.2 m/s (V_w); wind velocity pressure.

Evaluating the Crack's Resistance to Air Flow

We proceed by evaluating the "resistance" of the crack for the case considered i.e. "all types of windows and sliding glass doors". Reference is now made to Figure 2.6.

The static pressure at point o in Figure 2.6 is given by

$$P_o = (c^* \rho_o V_w^2 / 2) - \rho_o g h \qquad -(2.23)$$

where ASHRAE handbook (17) gives the wind pressure coefficient C* for elongated rectangular crack at T_a = 11.6°C as $C^* = 1.0$
ρ_o is the density of outdoor air (Kg/m^3);
Vw the wind speed (m/s); $V_w = 11.2$ m/s;
g the gravitational acceleration. g $= 9.81 m/s^2$,
and the distance between reference levels

and crack in metres, h. In an actual simulation, a crack such as the one in Figure 2.6, will represent all the cracks in an external wall of a room. This equivalent crack should be located halfway between floor and ceiling (18). Thus

$$h = 0.5\,H \qquad -(2.24)$$

where H is the height of the building.

The static pressure at point i is given by

$$P_i = P_v - \rho_i g h \qquad -(2.25)$$

where

P_{atm} is the inside pressure at reference level in (N/m^2) assumed atmospheric and ρ_i the density of indoor air (Kg/m^3). P_v (ref.17) is given as $(0.8C^{*}\rho_0 v_w^2/2) = 0.8C^{*}(1.2)(11.2)^2/2 = 60.2\ N/m^2$ ($C^{*} = 1.0$).

The density of air is given by the gas equation:

$$\rho_{air} = \frac{P_{atm}}{R_a T} \qquad -(2.26)$$

R_a is the universal gas constant for air and is R_a = 287.045 J/KgK air; P_{atm} is the atmospheric pressure in N/m^2, and T, the absolute temperature in (K). P = 1.01325 x 10^5 Pa (N/m^2).

From equations (2.23) to (2.26) the pressure difference ΔP in P_a of eqn. (2.21) can be written as

$$\Delta P = P_o - P_i = \frac{P_{atm}}{R_a}\left[\frac{0.2\,C^* v_w^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right] \qquad -(2.27)$$

where T_a is the outdoor ambient temperature, T_a = 11.6 °C; and T_c the indoor room temperature, T_c = 18.3°C.

From eqn. (2.21)

$$R_{oi}^{n} = \frac{(P_o - P_i)^n}{\dot{V}} \qquad -(2.28)$$

From eqn. 2.22 & 2.27, for V_w = 11.2 m/s; n = 0.66, C* = 1.0, the crack "resistance" $R_{o,i}$ is evaluated as

$$R_{oi} = \frac{P_o - P_i}{(\dot{V})^{1.5}} \qquad -(2.29)$$

$$R_{oi} = \frac{P_{atm}}{R_a}\,\frac{\left[\frac{0.2\,C^* V_w^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right]}{\dot{V}^{1.5}} \qquad -(2.30)$$

In using $T_{a,c}$ in subsequent calculations

$$T_{a,c} = T_a\,[K] \quad T_a > T_c \text{ (flow from o to i)} \qquad -(2.31)$$

and

$$T_{a,c} = T_c\,[K] \quad T_c > T_a \text{ (flow from i to o)} \qquad -(2.32)$$

From eqn. (2.30)

$$R_{oi} = \frac{\left[\frac{P_{atm}}{R_a}\right]\left[\frac{0.2\,C^*(11.2)^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right]}{\dot{V}^{1.5}} \qquad -(2.33)$$

where V_w = 11.2 m/s and $\dot{v}$ = 0.00254 m^3/s of A_o (ref.17).

Evaluating the Air Flow due to Infiltration. $\dot{m}_I$

To evaluate the weight flow (air leakage/infiltration rate) in Kg/s m^2 of A_o, we use eqn. (2.29) and (2.30). From these two equations, $\dot{m}_I = \rho \dot{V}$ is given by (in Kg/s m^2 of crack area A).

$A_o = \Sigma(A_i)$ the inlet area = (length x clearance of windows) x no. of window lengths involved. Then

$$\dot{m}_I = \left[\frac{\left[\frac{0.2C^* v_w^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right]\frac{P_{atm}}{R_a}}{\dfrac{\left[\frac{0.2C^*(11.2)^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right]\frac{P_{atm}}{R_a}}{(0.00254)^{1.5}}} \right]^{0.66} \quad -(2.34)$$

For a crack area, A_o, the weight flow or air leakage/infiltration rate in Kg/s is then given by

$$\dot{m}_I = A_o \times \frac{P_{atm}}{R_a T_{a,c}} \left[\frac{\left[\frac{0.2C^* v_w^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right]\frac{P_{atm}}{R_a}}{\dfrac{\left[\frac{0.2C^*(11.2)^2}{2T_a} - 0.5gH\left(\frac{1}{T_a} - \frac{1}{T_c}\right)\right]\frac{P_{atm}}{R_a}}{[0.00254]^{1.5}}} \right]^{0.66} \quad -2.35$$

In evaluating the infiltration rate from eqn. (2.35) above the wind pressure coefficient C* depends on the atmospheric temperature as well as wind velocity V_w. For T_a = 11.6 °C and a windspeed V_w = 11.2 m/s C^* is obtained as $C^* = 1.0$

For various values of T_a in (K); see ASHRAE handbook and Product Directory (17).

$$\frac{P_{atm} \cdot C^*}{2 R_a T_a} = 0.601 \qquad -(2.36)$$

C^* must be evaluated from eqn.(2.36) for a given ambient temperature; before us in eqn. (2.35), with T_a in ($^\circ K$). V_w in the numerator of eqn. (2.35) is not 11.2 m/s, but the pertinent value at the location.
H = height of building.

Sample Calculation

1. $R_{o,i}$: Crack "resistance".

Substituting T_a = 11.6 $^\circ$C (284.6K) in eqn. 2.36 gives C* as

$$c^* \simeq 1.0 \approx 0.96$$

For V_w = 11.2 m/s. *This is not the input for V_w in the numerator of eqn. (2.35) for $\dot{m}_I$, but the pertinent speed must be used.

$g = 9.81 \ m/s^2$

$R_a = 287.045 \ J/Kg\,K$ air

$\dot{V} = 0.00254 \ m^3/s$ of A_o

$T_c = 18.3^\circ C = 291.3 \ K$ (fixed)

$T_a = 11.6^\circ C = 284.6 K$ (Location value, check c^* eqn 2.36)

$T_{a,c} = T_c \ (T_c > T_a) = 291.3$ check eqn 2.31 / 2.32

$P_{atm} = 101325 \ Pa \ (N/m^2)$

$H = 4m$

Substituting the above values into eqn. (2.33) yields the "resistance" offered by crack to air flow as

$$R_{oi} = \frac{\left[\frac{P_{atm}}{R_a}\right]\left[\frac{0.2 \times 1.0 \times (11.2)^2}{2 \times 284.6} - \frac{9.81}{2} H \left(\frac{1}{284.6} - \frac{1}{291.3}\right)\right]}{(0.00254)^{1.5}} \quad -(2.37)$$

$$R_{oi} = \left[344.3 + 3.1\,H\right]\left[\frac{P_{atm}}{R_a}\right] \quad m^3/s\,m^2 \text{ of } A_o \quad -(2.38)$$

For a building of height H = 4m,

$$R_{oi} = \left[344.3 + 12.4\right]\left[\frac{101325}{287.045}\right] \quad m^3/s\,m^2 \text{ of } A_o \quad -(2.39)$$

Or

$$R_{oi} = 1.3 \times 10^5 \quad m^3/s\,m^2 \text{ of } A_o \quad -(2.40)$$

Eqn. (2.39) indicates that as the height of the building, H, increases, the crack "resistance" to air flow, R_{oi}, increases. This is to be expected as the increased gravitational forces in a taller building should increase the "resistance" to air flow by the crack.

R_{oi} can be plotted as a function of the building height H in (m)

2. Calculate $\dot{m}_I$ Eqn. (2.35)

From eqn. (2.35) assume a wind speed of 5 m/s. (The pertinent value must be used in the numerator of eqn. (2.35). Hence,

$$v_w = 5\,m/s$$

Assume

$H = 4m$

$P_{atm} = 101325\ Pa$ (same)

c^* (as calculated by eqn 2.36) using T_a [°K]

R_a (same) = 287.045 J/Kg K air.

$T_a = 11.6°c$ (check c^*) eqn 2.36 $c^* \approx 1.0$

$= 284.6°K$

T_c (Fixed) = 18.3°c [291.3°K]

Then from eqn. 2.35 for $\dot{m}_I$ and 2.38 for R_{oi}

$$\dot{m}_I = A_o \times \frac{P_{atm}}{R_a T_{a,c}} \left[\frac{\left[\frac{0.2 \times 1.0 \times (5)^2}{2 \times 284.6} - 0.5 \times 9.81 \times 4 \left(\frac{1}{284.6} - \frac{1}{291.3} \right) \right] \frac{P_{atm}}{R_a}}{1.3 \times 10^5} \right]^{0.66}$$

where

$$T_{ac} = \begin{cases} T_c & (T_c < T_a) \\ T_a & (T_a > T_c) \end{cases}$$

In this case $T_{a,c} = T_c = 291.3K$ ($T_c > T_a$).
Then,

$$\dot{m}_I = 1.2\, A_o \left[[6.8 - 1.2] \times 10^{-8} \times \frac{101325}{287.045} \right]^{0.66}$$

$$\dot{m}_I = 1.2\, A_o\, [7.9 \times 10^{-4}] = 9.5 \times 10^{-4}\ Kg/s\ m^2 \text{ of flow area.}$$

Assuming that the glazing area constitutes 3 windows/doors of size (1x1.5 m) and clearance 10^{-2} m; then the leakage area/flow area A_o is given by

$$A_o = 3[(1 \times 10^{-2}) \times 2 + (1.5 \times 10^{-2}) \times 2]$$

$$A_o = 3[0.02 + 0.03] = 3 \times 0.05$$

$$A_o = 0.15\ m^2$$

From which $\dot{m}_I$ becomes $\dot{m}_I$ = 0.00014 Kg/s or 1.4×10^{-4} Kg/s air

Hence the infiltration or air leakage rate

$$\dot{m}_I = 9.4 \times 10^{-4}\ \text{Kg/s air m}^2 \text{ of flow area.} \quad -(2.42)$$

This value of $\dot{m}$ is now used in eqn.(2.18) for T_{co}

$$T_{co} = \left[\left[\frac{F_R U_{eq}\left[(BF_c Q_T + (1-B) F Q_T)/U_{eq}\right]}{F_R U_{eq} + \dot{m} C_p}\right] + T_a\right] \quad -(2.43)$$

where $\dot{m}$ is as calculated above, i.e.

$$\dot{m}_{infil} = 9.4 \times 10^{-4}\ \text{Kg/s m}^2 \text{ of air flow area.} \quad -(2.44)$$

2.2.2 Rate of Ventilation Air Flow into Building, $\dot{m}_{vent}$, by (A) wind and (B) Thermal Forces.

It was pointed out in section 2.2.1 that, when natural ventilation is employed in a building, i.e. the occupants desire additional comfort and open windows and doors, infiltration flow is superseded and the term, $\dot{m}$, in the eqn. (2.43) above for T_{co} is now $\dot{m}_{vent}$ for the building envelop, which we evaluate, following recommendations in the ASHRAE fundamentals Handbook (17).

Air movement into, through or out of a building may be caused by wind or thermal forces, acting alone or together. Arrangement, location and control of ventilating openings should be to combine rather than oppose the two forces. In the following section the air flow in (m^3/s) due to

(a) Wind and

(b) Thermal Forces

shall be evaluated for our building envelop.

(a) Flow due to Wind

Factors affecting ventilation wind forces include average velocity v_w, prevailing direction, seasonal and daily variation in velocity and direction, and local obstructions such as nearby buildings or hills.

Wind velocities are usually lower in summer than in winter; frequency, from different directions differ in summer and winter. There are few places where velocity often falls below half the average. Thus, natural ventilating systems are often designed for wind velocities of half the average seasonal velocity, i.e.

$$v_w = \frac{1}{2}\,\bar{v}_{season} \qquad \text{— (2.45)}$$

where V_ω is the design velocity and $\bar{v}_{season}$ is the average seasonal velocity (120 days).

Eqn. (2.4 6) below shows the quantity of air forced through ventilation inlet openings by wind, or determines the proper size of openings to produce given air flow rates.
In (m^3/s),

$$\dot{V}_{wind} = Z A V_w (10^{-3}) \qquad -(2.46)$$

where $\dot{V}_{wind}$ is the air flow in (m^3/s): A is the free area of inlet openings in square metre; V_w is the design wind velocity given by eqn. (2.45) in m/s ; and Z the effectiveness of the opening. (Z is taken as 0.50 to 0.6 for perpendicular winds and 0.25 to 0.35 for diagonal winds (17)) i.e.

$0.5 \leq Z \leq 0.6$; perpendicular winds — (2.47)

$0.25 \leq Z \leq 0.35$; diagonal winds — (2.48)

In these computations the effectiveness of the opening will be taken as

$$Z = 0.5 \ (50\%)$$

Inlets should be directly into the most frequent prevailing wind direction. If they are not advantageously placed, flow will be less than that from the equation; if unusually well-placed, flow will be slightly more. Desirable outlet locations are

(1) On the leeward side of the building directly opposite the inlet.

(2) On the roof, in the low pressure area caused by jump of the wind.

(3) On the sides adjacent to the windwards face where low pressure areas occur.

(4) In a monitor on the leeward side.

(5) In roof ventilators; or

(6) By stacks.

(b) Flow due to Thermal Forces

If there is no significant building internal resistance, and assuming indoor and outdoor temperatures are close to 27°C (80°F), the flow in m^3/s, due to stack effects is

$$\dot{V}_{stack} = 89\,A\,(\Delta y\,(T_c - T_a))^{1/2} \times 10^{-3} \quad \text{For } (z = 50\%) \qquad \text{— (2.49)}$$

where $\dot{V}_{stack}$ is the air flow in (m^3/s);
A, the free area of inlets or outlets (assumed equal) in square metres;
Δy the height from inlets to outlets in metres and may be assumed

$$\Delta y = 0.5\,H \qquad \text{— (2.50)}$$

where H is the height of the building for a window located midway between floor and ceiling, assuming outlets employed are stack outlets on roof as in Fig.2.2 . T_c is the average indoor temperature in (°K). 89 in equation (2.49) is the constant of proportionality, for an effectiveness of opening Z = 0.5. T_a, ambient temp. in (°K).

Greatest flow per unit area of opening is obtained when inlet and outlet areas are equal. Eqn. (2.46) and (2.49) are based on their equality (≡ Area of glazing, window or door. Increasing the outlet area over inlet area, or vice versa, will increase air flow but not in proportion to added area. When openings are unequal, the usual practice is to use the smaller area in the equation and add the increase in flow percentage) as determined from standard curves. Reference is made to the ASHRAE handbookof fundamentals (17).
% increase from A16, yields the following fits.

(i) Flow due to wind forces, or thermal forces (Increase)

$$\partial \dot{V}_{stack} \; (\partial \dot{V}_w) = \frac{24.15}{100} \left(\frac{A_{out}}{A_i} - 1 \right)^{0.37} \quad - (2.51)$$

$r = 0.94.$ (percentage)

where A_{out} and A_i are the outlet area (stack area in roof), and inlet area (area of window or door in conservatory). The correlation coefficient for this fit is obtained as $r = 0.94$. Actual values of ($\Delta \dot{V}_w$) $\Delta \dot{V}_{stack}$ obtained from the curve in ref. (17) and those generated by the fit in equation (2.51) are presented in tables 2.1$ Fig 2.10, for various ratios of (A_{out}/A_i), i.e. outlet to inlet flow area.

When one uses equation (2.51) and equations (2.45) - (2.50), $\dot{V}_{wind}$ and $\dot{V}_{stack}$ are then evaluated in terms of the ratio of outlet to inlet area A_{out} / A_i. $\dot{V}_{wind}$ the air flow by ventilation due to wind forces in m^3/s is then from eqns (2.51) and (2.45) - (2.48).

$$\dot{V}_{wind}(m^3/s) = 0.5 \times 10^{-3} \frac{\bar{V}_{season}}{2} A \left[1 + \frac{24.15}{100} \left(\frac{A_{out}}{A_i} - 1 \right)^{0.37} \right] \quad -(2.52)$$

Using an effectiveness of opening of 0.5, and the wind velocity v_w as the average season value ($\bar{v}_{season}/2$)(m/s).

(ii) Mass flow rate of air due to wind <u>forces (Ventilation)</u> $\dot{m}_{vent,wind}$

From equation (2.52), the air mass flow rate by ventilation due to wind forces on open windows/doors simply becomes

$$\dot{m}_{vent,wind} = \rho \dot{V}_{wind} \ (Kg/s) \quad -(2.53)$$

and $\dot{V}_{wind}$ is given by eqn (2.52). Hence,

$$\dot{m}_{vent,wind} = \left[\frac{P_{atm}}{R_a T_{a,c}} \frac{\bar{V}_{season}}{2} 0.5 \times 10^{-3} A \right] \left[1 + .2415 \left(\frac{A_{out}}{A_i} - 1 \right)^{0.37} \right] \ (Kg/s) \quad -(2.54a)$$

where

$$A = \begin{cases} A_{out} \ (A_{out} < A_i) \ m^2 \\ A_i \ (A_{out} > A_i) \ m^2 \end{cases} \quad -(2.54b)$$

$$T_{a,c} = \begin{cases} T_a \ (T_a > T_c) \ ^\circ K \\ T_c \ (T_a < T_c) \ ^\circ K \end{cases} \quad -(2.54)c$$

$$\frac{\bar{v}_{season}}{2} = \text{mean wind velocity } v_w \text{ (m/s)}$$

— (2.54d)

P_{atm} is the atmospheric pressure, (P_{atm} = 101325 Pa)

and R_a is the universal gas constant for air, (R_a = 287.045 J/Kg K air).
T_c is the room temperature set at (T_c = 18.3°c [291.3°K])
and T_a the ambient temperature.

T_a the average heating season value ref.(2) is T_a = 11.6°c [284.6 K]

(iii) <u>Mass flow rate of air due to thermal forces (Ventilation)</u>

$\dot{m}_{vent, stack}$

Similarly from equations (2.51) and (2.49)-(2.50), the ventilation air mass flow rate, due to thermal forces (stack effects), (i.e. temperature differences between building envelop and ambient);

$$\dot{m}_{vent, stack} = \left[\frac{P_{atm}}{R_a T_{ac}} \times 10^{-3}\, 89A \sqrt{0.5H(T_c - T_a)^*}\right]\left[1 + .2415\left(\frac{A_{out}}{A_i} - 1\right)^{0.37}\right]$$

(kg/s)

—(2.55a)

$$(T_c - T_a)^* = T_a - T_c \quad (T_c < T_a) \quad -(2.55b)$$

where

$$A = \begin{cases} A_{out} & (A_{out} < A_i) \; m^2 \\ A_i & (A_{out} > A_i) \; m^2 \end{cases} \qquad -(2.55c)$$

$$T_{a,c} = \begin{cases} T_a & (T_a > T_c) \; {}^\circ K \\ T_c & (T_a < T_c) \; {}^\circ K \end{cases} \qquad -(2.55d)$$

P_{atm} is the atmospheric pressure (P_{atm} = 101325 Pa (N/m^2)); and R_a the universal gas constant for air, (R_a = 287.045 J/KgK air). The room temperature (T_c = 18.3°C (291.3K)); T_a the average heating season (120 days) ambient temperature is from ref.(2) T_a = 11.6°C (284.6K). H is the height of the building in (m).

(c) Ventilation mass flow rate due to Combined wind and stack effects

When both wind and thermal forces act together, even without interference, as is usually the case, resulting air flow is not equal to the two flows estimated separately. Flow through any opening is proportional to the square of the heads acting on that opening.

Wind velocity and direction, outdoor temperature and indoor distributions cannot be predicted with certainty, and refinement in calculation is not justified. A simple method (19) is to calculate the sum of the flows produced by each force separately.

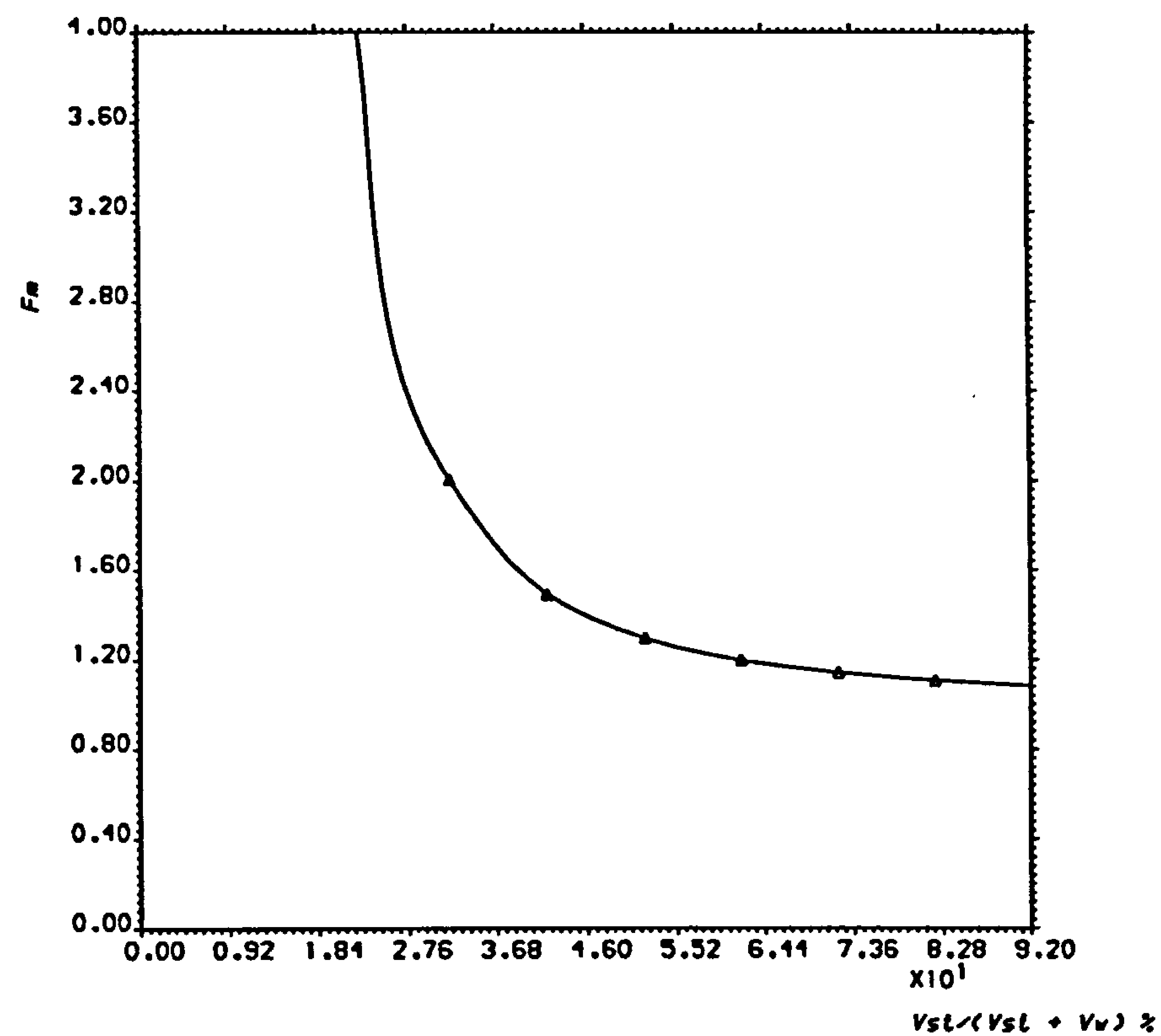
Then using the ratio of the flow produced by thermal forces to the aforementioned sum, the actual flow due to the combined forces can be approximated from standard curves (17) as a multiple of flow due to temperature difference. A fit on this curve, see A17, yields the actual flow as multiple of flow due to temperature difference; a function of the ratio

$$\dot{m}_{vent,stack} / (\dot{m}_{vent,stack} + \dot{m}_{vent,wind})$$

The multiplication factor on $\dot{m}_{vent,stack}$ that yields the combined air mass flow rate $\dot{m}_{vent,stack,wind}$ is empirically evaluated in A17. The air mass flow rate in (Kg/s) due to combined ventilation, wind and thermal forces can be written as

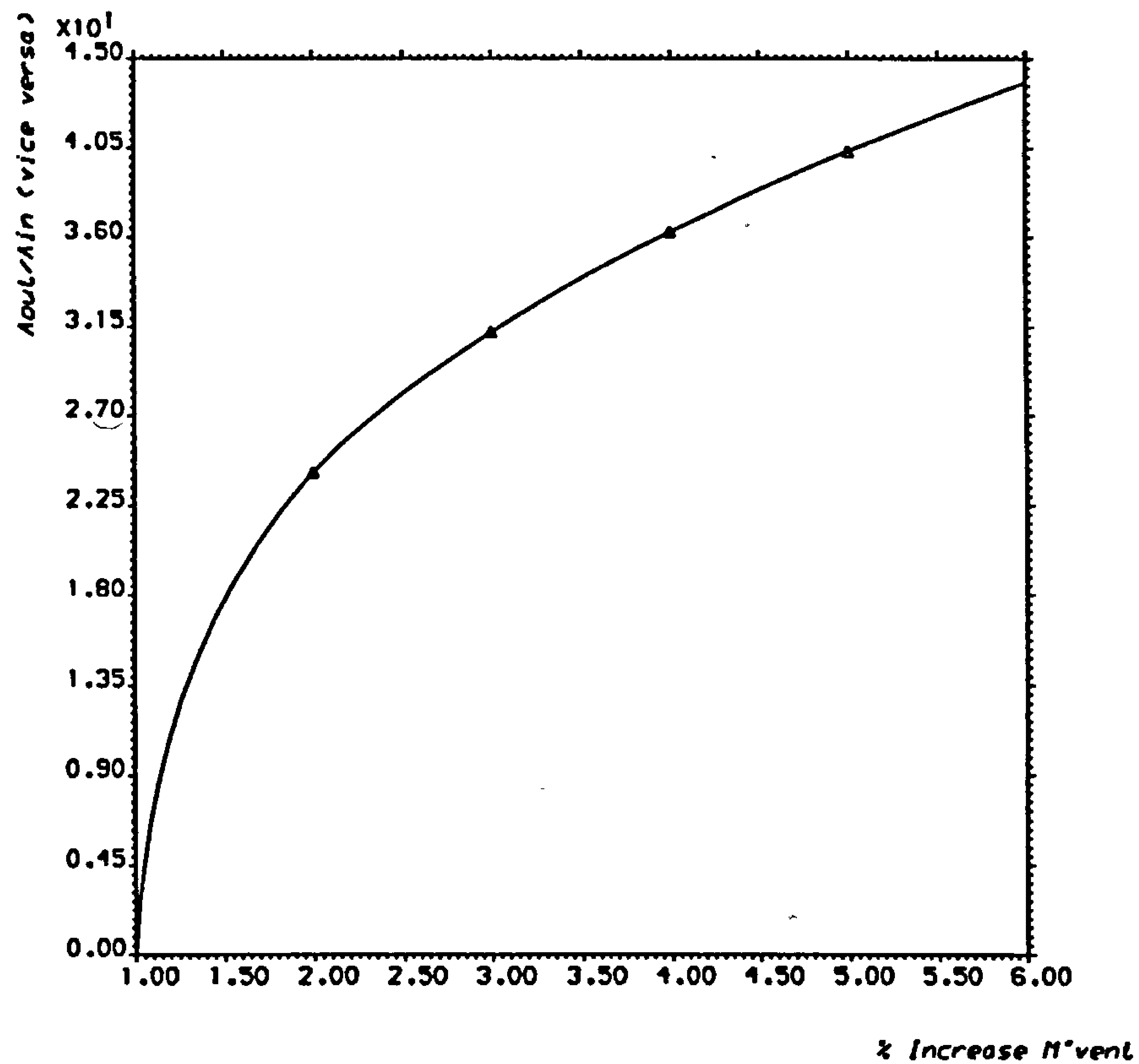
$$\dot{m}_{vent,wind,stack} = \left[\frac{355.5}{\left[100\left[\frac{\dot{m}_{vent,stack}}{\dot{m}_{vent,stack} + \dot{m}_{vent,wind}} - 0.1\right]\right]^{1.9}} + 1 \right] \dot{m}_{vent,stack} \qquad -(2.66)$$

A correlation of $r = -0.97$ makes such an empirical form attractive where $\dot{m}_{vent,stack}$ & $\dot{m}_{vent,wind}$ are the ventilation air mass flow rates in (Kg/s) due to thermal and wind forces and are given by equations (2.55a-d) and (2.54a-d) respectively.



Combined flow due to wind and temperature
forces as mult. of flow by temp. diff. only

Fig 2.9



Ventilation flow increase due to unequal inlet outlet area

Fig 2.10

Thus, when ventilation is activated in the building by opening windows and doors by the occupants, the combined ventilation air mass flow rate into the building is given by equation (2.66) above. Infiltration effects being thereby superseded and vice versa. The eqn. (2.18) for conservatory temperature then becomes

$$T_{co} = \left[\left[\frac{F_R U_{eq} \left[(B F_c Q_T + (1-B) F Q_T) / U_{eq} \right]}{F_R U_{eq} + \dot{m} c_{p\,vent,wind,stack}} \right] + T_a \right] \quad -(2.67)$$

where the heat removal factor is from eqn. (2.11)

$$F_R = \frac{\dot{m} c_{p\,vent,stack,wind}}{U_{eq}} \left[1 - \exp\left[-(U_{eq} / \dot{m} c_{p\,vent,stack,wind}) \right] \right] \quad -(2.68)$$

and the equivlant heat transfer coefficient for the building envelop U_{eq} from eqn. (2.25) is

$$U_{eq} = (1-B) U_{sa} + B U_{co,a} \quad -(2.69)$$

In equations (2.67)-(2.69), T_{co} is the conservatory temperature in (°K).

B the volumetric proportion of air (from unity) retained by the conservatory. (1 - B)is retained by room air at temperature T_c fixed at (T_c = 18·3°C (291·3 °K)). The fraction of transmitted insolation (Q_T = 1·224 × 10⁹ J/m² (2)) not absorbed by conservatory and storage in room; which directly heats the air in the conservatory and room are represented as F_c and F respectively.

T_a is the ambient temperature, $T_a =$ 11.6°C (284.6K); $\dot{m}$ vent, stack, wind is the ventilation air mass flow rate in (Kg/s) due to combined wind and thermal forces as in eqns (2.54a-d), (2.55a-d) and eqns. (2.66). C_p is the specific heat capacity of air at constant pressure, $P_{atm} =$ 101325 (Pa or N/m²), C_p = 1012 J/KgK. F_R is the equivlant heat removal factor for the building envelop (ratio, dimensions). U_{eq} is the equivalent heat transfer coefficient for the building ($W/m^2 K$). L is the length of the direct gain system in the flow direction (eqn. 2.12), (L = conservatory depth + height of room)(m). $U_{co,a}$, heat transfer coefficient from conservatory to ambient ($U_{co,a} = 3.0\ W/m^2 K$). U_{SA}, heat transfer coefficient from storage at room air, assumed $3 \times U_{co,S}$; i.e $U_{SA} = 17$ ($W/m^2 K$). U_{Sa}, heat transfer coefficient from room air to ambient, $U_{Sa} = 2.84\ W/m^2 K$ [ref 1]. $\dot{m}$ (vent, stack) is the ventilation air mass flow rate in (Kg/s) due to thermal forces as given by eqn. (2.55a-d), and $\dot{m}$ (vent, wind) the ventilation air mass flow rate in (Kg/s) due to wind forces as given by eqn.(2.54a-d).

Sample Evaluation of Ventilation air mass flow rate (Kg/s) due to combined wind and Stack Effects

1. Air Mass flow rate due to Stack effects ($\dot{m}_{vent,\,stack}$)

Equation (2.55a-d) gives

$$\dot{m}_{vent,\,stack} = \frac{101325 \times 89 \times 0.75\,(0.5 \times 4 \times 6.7)^{\frac{1}{2}}}{287.045 \times 291.3} \times 10^{-3} \times 1.36$$

Kg/s

$\dot{m}_{vent, stack} =$ 0.403 (Kg/s) = 0.403/0.75 =

0.53 Kg/s m² of exit area.

where;

assuming inlet area is four times outlet area, A_{out}/A_i or vice versa assumes a numerical volume of $4 \cdot (T_c - T_a)$ for $(T_c > T_a)$ eqn 2.55b is for $T_c = 18.3°C$ (291.3 K) and $T_a = 11.6°C$ (284.6 K); $T_c - T_a = 6.7$ K. For a height of building H = 4 m (Eqn 2.55b). Eqn 2.55c yields $A = A_{out} = 0.25 \times 3 = 0.75 m^2$, assuming $A_i / A_{out} = 4$ and A_i constitutes 3 windows (1 x 1.0 m) and 1 cm clearance. Then $A_i = 3.0 m^2$ and from $A_{out}/A_i = 0.25$, we obtain $A_{out} = 0.75 m^2$.

2. <u>Air Mass flow rate due to wind effects</u>

Equation (2.54a-d) gives

$$\dot{m}_{vent, wind} = \frac{101325 \times 0.5 \times 0.75 \times 5 \times 10^{-3} \times 1.36}{287.045 \times 291.3}$$

$\dot{m}_{vent, wind} =$ 0.0031 (Kg/s) = 0.0031/0.75

= 0.0041 Kg/s m² of exit area.

assuming the same conditions in 1 above including an average wind speed $\bar{v}_w = 5$ m/s i.e $\bar{v}_{season} = 10$ m/s.

From the above example, the increase in ventilation rate due to unequal outlet inlet area ($A_i/A_{out} = 4$) and an inlet area of 3 windows (1 x 1.0m), A_i = 3x1.0m is 36%. (1.36 term)
It is apparent too that the stack ventilate rate > by wind forces $\dot{m}_{vent,stack}$ = 0.403kg/s, $\dot{m}_{vent,wind}$ = 0.0031 kg/s for a wind speed of v_{wind} = 5 m/s and air of density 1.2 Kg/m³.

3. Air Mass Flow Rate due to Wind and Stack Effects

From the sample calculations 1 and 2 it is apparent that

$$\frac{\dot{m}_{vent,stack}}{\dot{m}_{vent,stack} + \dot{m}_{vent,wind}} = 99\,\%$$

i.e. th stack ventilation air mass flow rate is 99% of the combined flow rate by wind and stack forces.

The combined air flow rate is not simply the sum of the two flow rates but evaluated from equation 2.66 above. Hence,

$$\dot{m}_{vent,wind,stack} = \left[\frac{355.5}{\left[100\left[\frac{0.403}{0.403+0.0031} - 0.1\right]\right]^{1.9}} + 1 \right] \dot{m}_{vent,stack}$$

$$= \left[\frac{355.5}{\left[100\left[0.99 - 0.1\right]\right]^{1.9}} + 1 \right] \dot{m}_{vent,stack}$$

$$= (1.07)\, \dot{m}_{vent,stack}$$

Therefore, the combined ventilation air flow rate for the above example is equivalent to a 7% increase on $\dot{m}$ (vent, stack) for a flow where stack effects predominate by 99% of the combined flow, and we obtain

$$\dot{m}_{vent, wind, stack} = 1.07 \times 0.403 \text{ Kg/s}$$

OR

$$\dot{m}_{vent, wind, stack} = 0.431 \text{ (Kg air/s)}$$

when ventilation supersedes infiltration, this is the value of $\dot{m}$ in the eqn. for T_{co} and vice versa.

(d) Practical Considerations when Utilising Natural Ventilation

Types of natural ventilation openings include

(1) windows, doors, monitor openings and skylights.

(2) Roof ventilators.

(3) Stacks connecting to registers; and

(4) Specially designed inlet or outlet openings.

Windows

Windows transmit light, of course, and provide ventilating areas when open. They may open by sliding vertically or horizontally; by tilting on horizontal pivots at or near the centre; or by swinging on pivots at the top, bottom or side. Air flow per square metre of opening may be considered the same. Type of pivoting is an important consideration for weather protection.

Roof Ventilators

A roof ventilator provides a weather-proof air outlet. It is actuated by the same forces that create flow through other openings. Its capacity depends on its location on the roof; the resistance it and the ductwork offers to air flow; its ability to use kinetic wind energy to induce flow by centrifugal or ejector action; and the height of the draft.

A ventilator should be located on that part of the roof where it receives the full wind without interference. If it is installed within the suction region created by the wind passing over the building in a high court, or on a low building between two high buildings, its performance will be seriously impaired and its ejector action, if any, may be lost.

For a high flow coefficient, the ventilator base should be of taper cone design to produce a bell-mouth nozzle. If a grille is provided at the base, or structural members introduce resistance, the base opening should be increased accordingly.

Building air inlets at lower levels should be larger than the combined throat areas of all roof ventilators.

Natural draft, or gravity, roof ventilators may be stationary, pivoting, or oscillating, and rotating. Selection criteria are: ruggedness; corrosion-resistance; stormproofing features; dampers and operating mechanisms; possibility of noise; original cost; and maintenance. Natural ventilators can supplement power driven supply fans; the motors need only be energised when the natural exhaust capacity is too low. Gravity ventilators may have manual dampers or dampers controlled by thermostat or wind velocity.

Stacks or vertial flues should be located where wind can act on them from any direction. Without wind, the chimney effect alone removes air from the room with the inlets.

Generally, the following natural ventilation rules must be considered.

1. Buildings and ventilating equipment should not be oriented for a particular wind direction, but should be designed for effective ventilation with all wind directions, since very few places have prevailing winds. Even if a wind direction prevails 80% of the time, safe adequate ventilation is needed for the remaining 20%.

2. Inlet openings should not be obstructed by buildings, trees, sign boards, or indoor partitions.

3. Greatest flow per unit area of total opening is obtained by using inlet and outlet openings of nearly equal areas. See Fig 2.7.

4. Direct short circuits between openings on two sides at a high level may clear the air at that level without producing any appreciable ventilation at the lower level of occupancy.

5. For temperature difference to produce a motive force, there must be vertical distance between openings; vertical distance between inlets and outlets should be as great as possible.

6. Openings in the vicinity of the neutral presssure level are least effective for ventilation, and

7. Openings with areas much larger than calculated are sometimes desirable, when anticipating increased occupancy, or very hot weather. The openings should be accessible to and operable by occupants.

2.3 Solar-Load-Ratio Statistics in the United Kingdom

The original method of Gordon and Zarmi (1,2) did obtain close form analaytic expressions for the solar heating fraction (SHF) as a function of the solar load ratio. The analysis as presented by Gordon and Zarmi did assume the frequency distribution of daily SLR values to be parabolic about the mean. It was however recommended that the actual SLR frequency distribution for the location be employed in estimating the SHF values. Cowing and Kreider (20), in their study of locations in the United States, obtained the actual frequency distribution of SLR values for twenty-six cities in the United States. They showed that a fourth order polynomial proved sufficiently accurate in characterising all SLR values. The relationships was of the form.

$$p(SLR_q) = A_g + B_g SLR_g + C_g SLR_g^2 + D_g SLR_g^3 + E_g SLR_g^4 \qquad (2.70)$$

Higher order expressions increased the complexity of the closed-form solar fraction expressions with only marginal increase in accuracy (21). Basically, the work in (20) presented the following improvements on the original Gordon, Zarmi (G,Z) method.

1. New SLR distribution functions for 26 United States Typical Meteorological Year (TMY) locations (22). Improvements on the original GZ SLR distribution was felt to be warranted since discrepancies in annual solar functions up to 25 per cent were found in climates with skewed SLR distributions. (21).

2 The derivation of a generalised, closed-form expression for annual system performance incorporating these new SLR distribution functions.

3. Additional climate and location related inputs to further assist the system designer, and

4. Comparison of measured auxiliary energy usage in passively heated, solar building with predictions of the improved model. The generic SLR_g, was defined as in eqns 2.71 – 2.73 i.e

$$SLR = \frac{\bar{LD}}{\bar{L}} = \frac{A_c \bar{\tau\alpha} I_v}{24 \times \underbrace{UA}_{L_{c,i}\ (\text{Build. Loss. Cft.})} (T_c - T_a)_{\text{hrly DD}}}$$

$$= \frac{A_c \bar{\tau\alpha}}{L_{c,i}} \cdot \frac{I_v}{24\, DD(\text{hrly.})}$$

Then if,

$$SLR_g = I_v / 24\, DD(\text{hrly}) \qquad \text{— } (2.71)$$

and

$$G = L_{c,i} / A_c \bar{\tau\alpha} \qquad \text{— } (2.73)$$

$$SLR = SLR_g / G \qquad \text{— } (2.72)$$

where I_{v} is the total daily solar radiation impinging upon a south facing vertical glazing (Typical Meteorological Year Data). DD_d is the degree day using 65°F base. A_c the collector area; $\bar{\tau}$, the average yearly transmittance of the solar glazing $\cdot\alpha$, the absorption of the direct gain passive solar space heating system to transmitted radiation, and $L_{c,i}$ (UA value), the building loss coefficient (heat transfer coefficient) attributable to conduction and infiltration.

As in (1,2), the annual solar fraction is given by

$$SHF_{ann} = F_{a,d}\int_{SLR_{min,d}}^{\infty} p(x)\,dx + \int_{0}^{SLR_{min,d}} p(x)\,SHF_d(x)\,dx + \bar{F}_{a,n}\int_{SLR_{min,n}}^{\infty} p(x)\,dx + \int_{0}^{SLR_{min,n}} p(x)\,SHF_n(x)\,dx \quad -(2.74)$$

substitution of equation (2.71)-(73) into equation (2.70) yields, noting $A_g = G\cdot A$ & $SLR_g = GSLR$

$$p(SLR) = GA_g + B_g G^2 SLR + C_g G^3 SLR^2 + D_g G^4 SLR^3 + E_g G^5 SLR^4 \quad -(2.75)$$

The substitution of eqn. (2.75) into the first two terms of eqn. (2.74) yields the annual performance eqn. as

$$\begin{aligned} SHF_{ann,d} &= A_g G Z + B_g G^2 Z^2/2 + C_g G^3 Z^3/3 \\ &\quad + D_g G^4 Z^4/4 + E_g G^5 Z^5/5 \\ &\quad - \Big[A_g G/2C_d + B_g G^2/6C_d^2 + C_g G^3/12 C_d^3 \\ &\quad + D_g G^4/20 C_d^4 + E_g G^5/30 C_d^5 \Big] \quad -(2.76) \end{aligned}$$

where Z is the maximum daily SLR value for a given location; Ag-Eg are constants and Cd, Cn are user calculated, constant of proportionality linking daytime, night time system performance to SLR respectively. Polynomial regression techniques (23) are utilised to find best fit values for Ag, Bg, Cg, Dg and Eg. The freq. distributions for three United States locations are shown in Figs 1-3 of ref(20), including a comparison of the Actual and Modelled SLRg distribution.

The sum of the first five terms of eqn.(2.76) is equal to unity, since it is the area under the normalised distribution curve. The remaining terms are simplified by defining seven new parameters.

$$V_d = G/C_d \ , \quad V_n = G/C_n \ , \quad A = A_g/2 \ , \quad B = B_g/6$$

$$C = C_g/12 \ , \quad D = D_g/20 \ , \quad E = E_g/30 \qquad \text{— (2.77)}$$

Then, bearing in mind the sum of the first five terms of eqn. (2.76) is unity the $SHF_{ann,d}$ of eqn. (2.76) can be written as

$$SHF_{ann,d} = 1 - [A + (B + [C + (D + E V_d) V_d] V_d) V_d] V_d \qquad \text{— (2.78)}$$

The expression for the night time SHF is identical in form except that V_d is replaced by V_n .

The terms A, B, C, D and E are tabulated by location in Table 1 of ref [20] for 26 USA locations.

The proceeding expressions are valid except for no-venting situations, i.e. those for which all solar radiation transmitted into the living heated space is useful in offsetting auxiliary energy space heating requirements (for example, in residences with relatively small south glazing areas and/or large building heat loss coefficients). For no vent cases, annual system performance is directly proportional to the average daily solar-to-load ratio (1,2). In equational form

$$SHF_{ann,d} = C_d \left[A_c \bar{\tau\alpha} / L_{c,i} \right] S = S/V_d \quad -(2.79)$$

$$SHF_{ann,n} = C_n \left[A_c \bar{\tau\alpha} / L_{c,i} \right] S = S/V_n \quad -(2.80)$$

where S, the average daily solar-to-load ratio, is

$$S = \sum_{i=1}^{H} SLR_{g_i} / H \quad -(2.81)a$$

$$\& \quad SLR_g = I_v(\text{daily}) / 24(DD)(\text{hourly}) - (2.81)b$$

The SHF value below which eqn. (2.78) is not to be used was determined for the three system types considered and found to be independent of system type and is listed in Table 1 (ref 20) as R. If either

$$S/V_d < R \quad \text{OR} \quad S/V_n < R \quad -(2.82)$$

then the annual solar fraction is given by eqn (2.79) or (2.80) respectively, rather than (2.78).

Finally, the total annual solar heating fraction for day and night periods is calculated from

$$SHF_{ann} = F_{a,d}\, SHF_{ann,d} + F_{a,n}\, SHF_{ann,n} \quad -(2.83)$$

and the annual auxiliary energy space heating requirement is

$$Q_{aux,a} = (1 - SHF_{ann})\, L_{ann} \quad -(2.84)$$

where $F_{a,d}$ and $F_{a,n}$ are the average fraction of the space heating load occurring during daytime/night time over the heating season.

$$F_{a,d} = \frac{UA\,(T_e - T_{a,d}) \times \text{day length (hrs)}}{UA\,(T_e - T_a)(\text{hrly}) \times 24(\text{hrs})} = \frac{\bar{L}_d}{\bar{L}}$$

or $F_{a,d} = (T/24)(18.3 - T_{a,d})/(18.3 - T_a)$ — (2.85)

T being the average daytime length over the heating season, $T_{a,d}$ and T_a being the average heating season daytime/daytime-nighttime temperature, and L_{ann} being the annual space heating load.

$$L_{ann} = L_{ci}\,A_c\,(18.3 - \bar{T}_a) \quad — (2.86)$$

The method described in the proceeding sections will be applied to U.K. locations. Typical Reference Year (TRY) data from the Polytechnic of Central London (10 yrs ref) for Kew, London will be analysed with a view to obtaining the SLR frequency distribution. It will be seen if this distribution is sufficiently described by a fourth order polynomial as for the United States locations, and the degree of skewness about the mean SLR value will be studied to see the derivation from a simulation employing a parabolic form.

The exact integration for obtaining eqn. (2.76) is presented below.

<u>The Approach of Cowing and Kreider</u>(20)

The SHF_d is given by eqn. 2.74 i.e.

$$SHF_d = \frac{\bar{L}_d}{\bar{L}} \left[\int_{1/c}^{z} p(x)\,dx + \int_{0}^{1/c} p(x)\,c\,x\,dx \right] \quad — (2.87)$$

The upper limits of the first integral (∞) of eqn. (2.74) is substituted by Z (the maximum daily SLR value occurring during the heating season).

Gordon and Zarmi (1) (eqn 7) show $SHF_d^{(0)}(x)$ the SHF for days during which back up is required can be given by

$$SHF_d^{(0)}(x) = cx \qquad -(2.88)$$

i.e. the term in the second integral and also

$$SLR_{min,d} = 1/c \qquad -(2.89)$$

i.e. the lower limits of the first integral and upper limits of the second integral of eqn. (2.74 and 2.87). Eqn. (2.75) for the frequency of occurrence distribution of SLR is now substituted into eqn (2.87) above to yield

$$SHF_d = \frac{\bar{L}_d}{\bar{L}}\left[\int_{1/c}^{Z}(GA_g + B_g G x + C_g G^2x^2 + D_g G^3x^3 + E_g G^4x^4)\,dx \right.$$

$$\left. + \int_{0}^{1/c}(GA_g + B_g G x + C_g G^2x^2 + D_g G^3x^3 + E_g G^4x^4)\,cx\,dx\right] \qquad -(2.90)$$

The integral is evaluated by considering similar terms in both integrals of SLR (x).

Ag Term

(First Integral) $$\int_{1/c}^{Z} (G \cdot A_g)\,dx = G A_g x \Big|_{1/c}^{Z} = A_g G Z - \frac{A_g G}{c} \quad -(2.91)$$

(Second Integral) $$\int_0^{1/c} G \cdot A_g C x = A_g G C \frac{x^2}{2}\Big]_0^{1/c} = \frac{A_g G}{2c} \quad -(2.92)$$

combining eqns (2.91) and (2.92) yield

$$A_g G Z - \frac{A_g G}{c} + \frac{A_g G}{2c}$$

or

$$A_g G Z - \frac{A_g G}{2c} \quad -(2.93)$$

This appears as the Ag term of eqn (2.76) for SHF_d .

Bg Term

1st Integral eqn (2.90) $$\int_{\frac{1}{c}\cdot G}^{ZG} B_g G x\,dx = B_g G \cdot \frac{x^2}{2}\Big|_{\frac{1}{c}\cdot G}^{ZG} = \frac{B_g G^2 Z^2}{2} - \frac{B_g G^2}{2c^2} \quad -(2.94)$$

2nd Integral $$\int_0^{\frac{1}{c}\cdot G} (B_g \cdot Gx) c x\,dx = \int_0^{\frac{1}{c}\cdot G} B_g \cdot G \cdot c x^2\,dx = \frac{B_g G c x^3}{3}\Big|_0^{\frac{1}{c}\cdot G}$$

$$= \frac{B_g G^2}{3c^2} \quad -(2.95)$$

Combining (2.94) and (2.95) yields

$$\frac{B_g G^2 Z^2}{2} - \frac{B_g G^2}{2c^2} + \frac{B_g G^2}{3c^2}$$

$$= \frac{B_g G^2 Z^2}{2} - \frac{B_g G^2}{6c^2} \quad -(2.96)$$

and this appears the Bg terms of eqn (2.76) for SHF_{d} .

Thus evaluation of the Cg, Dg and Eg terms of the integral (eqn. 2.90) combining the Z and subscripted yields

$$SHF_{d} = A_g GZ + B_g \frac{G^2Z^2}{2} + C_g \frac{G^3Z^3}{3} + D_g \frac{G^4Z^4}{4} + E_g \frac{G^5Z^5}{5} - \left[\frac{A_g G}{2C} + \frac{B_g G^2}{6C^2} + \frac{C_g G^3}{12C^3} + \frac{D_g G^4}{20C^4} + \frac{E_g G^5}{30C^5} \right] \quad -(2.97)$$

and the expression for SHF_{d} is obtained.

Performing the first substitution of the upper limits of integral 1 in eqn. (2.90) using $\rho(x)$ as given by eq. (2.75) yields the SLR (x) distribution or Z distribution as if with limits from (0 to SLR max) or (0 to Z) .

This of course is the area under the normalised distribution curve (and therefore the sum of the first five terms of eqn. 2.97 (Z - terms) is unity).

The remaining terms of eqn. 2.97 are simplifed by defining six new parameters.

$$V_{d} = G/C \qquad A = A_g/2 \qquad B = B_g/6$$

$$C = C_g/12 \qquad D = D_g/20 \qquad E = E_g/30 \quad -(2.98)$$

Then bearing in mind the sum of the Z terms of eqn. (2.97) are unity, the expression becomes

$$SHF_{d} = 1 - \left[A + \left(B + \left[C + \left(D + EV_{d} \right) V_{d} \right] V_{d} \right) V_{d} \right] V_{d} \quad -(2.99)$$

Thus from first principles we arrived at Cowing and Kreiders expressions for SHFd. Expressions for SHF_n are similar to (2.99) except V_d is replaced by V_n & $V_n = G/C_n$.

2.4 SHF Analysis for Milton Keynes Linford Houses

The simulated results gave

$$\rho(SLR) = 2.787217 + 441.568843x - 1750.04674x^2 + 2552.68965x^3 - 1600.924x^4 + 363.4545x^5 \qquad -(2.100)$$

The SHF_d is given by eqn (2.74) i.e.

$$SHF_d = \frac{\bar{L}_d}{\bar{L}}\Big[\int_{1/c}^{Z}(2.787217 + 441.568843x - 1750.04674x^2 + 2552.68965x^3 - 1600.924x^4 + 363.4545x^5)dx$$
$$+ \int_0^{1/c} cx(2.787217 + 441.568843x - 1750.04674x^2 + 2552.68965x^3 - 1600.924x^4 + 363.4545x^5)dx\Big] \qquad -(2.101)$$

2.787217 Term

(First Integral)

$$\int_{\frac{1}{c}}^{Z} 2.787217\,dx = 2.787217x\Big|_{1/c}^{Z} = 2.787217Z - \frac{2.787217}{c} \qquad -(2.102)$$

(Second Integral)

$$\int_0^{1/c} 2.787217\,cx\,dx = 2.787217c\frac{x^2}{2}\Big|_0^{1/c} = \frac{2.787217}{2c} \qquad -(2.103)$$

Combining eqn. (2.102) and (2.103) yields

$$2.787217Z - \frac{2.787217}{c} + \frac{2.787217}{2c}$$

or

$$\frac{\bar{L}_d}{\bar{L}}\left[2.787217Z - \frac{2.787217}{2c}\right]$$

The difference in the coefficients from those of Cowing and Kreider results from the fact that the G term left in their expressions was already introduced in the computer data. However, it can be seen the curves of $\rho(x)$ fit their results. Also a 5th order polynomial for $\rho(x)$ was used compared to their 4th order eqn. Now considering the other coefficients of eqn. (2.101) yields the expression for SHFd for the Linford houses as

$$SHF_d = \frac{\bar{L}_d}{\bar{L}}\Big[2.787217Z + 441.568843\frac{Z^2}{2} - 1750.04674\frac{Z^3}{3} + 2552.68965\frac{Z^4}{4} - 1600.924\frac{Z^5}{5} + 363.4545\frac{Z^6}{6} - \Big[\frac{2.787217}{2c} + \frac{441.568843}{6c^2} - \frac{1750.04674}{12c^3} + \frac{2552.68965}{20c^4} - \frac{1600.924}{30c^5} + \frac{363.4545}{42c^6}\Big]\Big] \quad -(2.104)$$

As before, the sum of the first five terms of eqn. (2.104) equal unity.

Note that the definition for ($V_d = 1/c$) differs from Cowing and Kreiders ($V_d = G/c$) as mentioned before.

Hence,

$$SHF_d = 1 - [A + (B + [C + (D + E V_d^{*}) V_d] V_d) V_d] V_d \quad -(2.105)$$

where

$A = 2.787217/2$

$B = 441.568843/6$

$C = -1750.04674/12$

$D = 2552.68965/20$

$E = -1600.924/30$

$F = 363.4545/42$

$E V_d^{*} = (E + F V_d)$

and

$V_d = 1/c$

Hence SHF

$$SHF = SHF_d + SHF_h \quad -(2.106)$$

The p(x) of H and D distribution for Kew, gardens London, is shown in Figs. 2.33.

CHAPTER 3

COMPUTER PROGRAM MODULES

File	Module	Description	Results File
JOBI.FOR	LOGAR	D.G. House	SLRLOLD.DAT2 SLRDL.DAT2 RTSLR.DAT2 EFSLR.DAT LRDOSLR.DAT2
LOGONE.FOR	LOGONE	D.G. + Con- servatory	SLRLOLD.DAT1 SLRDL.DAT1 RTSLR.DAT1 EFSLR.DAT1 LRDOSLR.DAT1
LOGTWO.FOR	LOGTWO	Conservatory Study	FFR.DAT5 – U_{eq} vs. T_{co} FFR.DAT6 – T_{co} vs. B FFR.DAT7 – U_{eq} vs. F_R. FFR.DAT8 – B vs. F_R. FFR.DAT1 – F_R vs. $\dot{m}c_p/U_{eq}$. FFR.DAT2 – M_i or M_v vs. T_{co}.

File	Module	Description	Result File
	PCLTRY.DAT	Hourly weather data. Kew Gardens	10 year record
	LINS.DAT	Hourly weather data, Linford Houses, M.K.	May 82 - April 83
NOW.FOR Kew, London	YNOTSLR1	Computes, (i) Daily SLR (Kew). Day no, SLR.	SLR.DAT
		(ii) Day no, DD_d, $I_v (J/m^2)$.	MA.DAT
		(iii) p(SLR) real, computed	DHS.DAT
		(iv) Matrix of intercorrelation x-wn 5 variables of Polynomial eqn.p(SLR)	R IJ.DAT
RLNOW.FOR Linford Houses	YNOTSLR2	Computes, (i) Daily SLR (Linford,MK) Day no, SLR	SLR1.DAT
		(ii) Day no, DD_d, $I_v (J/m^2)$.	OPT2.DAT2
		(iii) SLR(s) p-SLR(s)	DHS1.RES
		(iv) RIJ	RIJ.DAT

File	Module	Description	Results File
VECTOR.FOR	VECTOR	In the process of determining the cfts of the polynomial eqn p-SLR, the inverse transformation of the 4-D space vector (RIJ.DAT) is required. Program Vector was developed to handle the problem. It finds the inverse of a (3 x 3) or (4 x 4) matrix by Gaussian elimination. It could easily be modified to an nthD problem, with n specified by the user. 2 case examples have their results (3 x 3) and (4 x 4) in Files MATX3.DAT and MATX4.DAT.	MATX3.DAT MATX4.DAT
POLY.FOR	POLY	Program Poly uses the results of the Inverse Matrix from Program Vector in Files MATX4.DAT and MATX3.DAT. and via Polynomial Regression Techniques, obtains the cfts of p-SLR distribution.	REG.RES
RLNOW.FOR	YNOTSLR2	Compares weather data (Computed) p(SLR) vs the equation (computed p(SLR) for specified SLR values).	DHS1.DAT

File	Module	Description	Result File
CONVERT.FOR			
	CONVERT	As the Linford data exceeded the width of the Vax Terminal, Convert was developed to read all the available records and write the desired outputs in File LINS.DAT. Reads from File in FILEOUT.DAT	 LINS.DAT
QAUX.FOR (G - Z Approach) Gordon and Zarmi(1)	QAUX I	Studies auxilliary heating requirements for Linford House 33,35 & 36 using a parabolic p-SLR.	G1.DAT House 33 SHF vs SLR -
		QAUXM - Auxilliary measured energy for space heating. QAUXCGZ-computed G - Z -	G2.DAT1 QAUXM vs QAUXCGZ House 33
		G2.DAT SHF vs SLR (G - Z) House 35 -	- G3.DAT SHF vs SLR House 36 (G - Z) -

File	Module	Description	Result File
		G2.DAT2 QAUXM vs QAUXCGZ House 35	- G3.DAT3 QAUXM vs QAUXCGZ House 36 (G - Z)
		- G13.DAT13 QAUXM vs QAUXCGZ All 3 houses (G - Z)	- GS13.DAT13 SHF vs SLR All 3 houses (G - Z)
QAUXB.FOR (C K Approach) Cowing & Kreider Approach (20)	QAUXJ	Studies auxilliary energy requirements for Linford Houses 33,35 and 36 using a 5th order polynomial p-SLR. QAUXM - Auxilliary measured energy for space heat- ing (Kwh)/month QAUXCK - comput- ed CK (Kwh)/month - F3.DAT SHF vs SLR House 36(CK)	F1.DAT SHF vs SLR House 33(CK) - F1.DAT1 House 33(CK) QAUXM vs QAUXCK - F2.DAT SHF vs SLR House 35(CK)

File	Module	Description	Result File
		F3.DAT3 QAUXM vs QAUXCK House 36(CK)	F2.DAT2 QAUXM vs QAUXCK House 35(CK)
		- F13.DAT13 QAUXM vs QAUXCK All three houses(CK)	- - FS13.DAT13 SHF vs SLR All three houses (CK)

3.1 MODULE LOGAR (The Direct Gain House)

Module Logar is a precise package which has the capabilities of reproducing the following results, for a direct gain system.

1. Solar heating fraction (SHF) versus relaxation time (RT) at different values for solar load ratio (SLR). The outputs are presented in Figure (2.15).

2. The SLR Vs SHF at different values for total to daily load ($\bar{L}/\bar{L}_d$).

Figure (2.11) shows the resultant outputs.

3. The SLR Vs SHF at different lengths of daytime period (d_ℓ) of 8, 10 and 12 hours. Outputs in Figs. (2.12).

4. The SLR Vs SHF at different values for delta to average solar load ratio. Outputs in Figs. (2.14).

5. The fraction F of transmitted insulation through vertical double glazing (for fixed solar load ratios) versus the solar heating fraction (SHF). Outputs in Fig. (2.13).

3.2 PROGRAM MODULE LOGONE
(d.g. house plus conservatory)

In introducing the conservatory to the direct gain system PM-LOGONE was developed to generate the following results.

1. SHF vs SLR for a direct gain system with CONSERVATORY and night insulation T_{co} (289K) 17 $^{\circ}$c; B = 0.2; infiltration rate $M_i = 9.4 * 10(-4)$ Kg/s m^2.

   at various $\bar{L}d/\bar{L}$. The outputs are presented in Figs 2.16.

2. The SHF vs relaxation time Rt for a direct gain house with conservatory B = 0.2 at specified SLRs. Figs 2.18 present the outputs.

3. The SHF vs F (fraction of Qt not absorbed by storage), for a direct gain house with conservatory at specified SLRs of 0.1, 0.5, 1.0, and 2.0. Figs. 2.20 present the generated outputs.

4. The SHF vs FC (fraction of Qt not absorbed by storage in conservatory) at specified SLRs of 0.1, 0.5, 1.0 and 2.0. The outputs are presented in figs 2.19.

5. SHF vs SLR for a d.g. system + conservatory at values of $\partial / \overline{SLR}$ of 0.6, 0.8, and 1.0. Outputs in Figs. 2.21.

6. SHF vs SLR for a d.g. house with conservatory a fourth the size of house B = 20% = 0.2 and infiltration rate of $\dot{m}_I$ = 9.5E - 04 kg/s m^2 air at various lengths of daytime period (dℓ) of 8, 10 and 12 hours. Outputs in Figs. 2.17.

3.3 PROGRAM MODULE LOGTWO
(Conservatory Study)

It was then desired to study factors influencing parameters as the conservatory temperature T_{co}, the heat factor F_R, the air flow rate by infiltration $\dot{M}_I$, and air flow rate by ventilation $\dot{M}_V$. Module Logtwo was developed to consider the problem.

Logtwo studied the following:

1. Conservatory temperature for increasing heat transfer coefficient from envelop of building to ambient. The outputs are presented in Figs 2.27.

2. T_{co} Vs B (the size of the conservatory w.r.t (the building and unity). Outputs in Figs 2.28.

3. Effect of increasing U_{eq} the heat transfer coefficient for building envelop on heat removal factor F_R. Outputs in Figs 2.29.

4. Effect of larger conservatories on the heat removal factor F_R, (flow considered by infiltration). $\dot{M}_I = 9.5E-04$ Kg/sm^2 Outputs in Figs 2.30.

5. F_R Vs $\dot{m}_I C_p / U_{eq}$ at various sizes of conservatories (B).

6. T_{co} Vs mass flow rate into building envelop at various conservatory sizes, B. Outputs in Figs 2.31 Wind speed V_{wind} = 5 m/s

CHAPTER 4

DISCUSSION OF RESULTS

4.1 The Passive Solar Direct Gain House

Using the theory developed in sections 1 and 2, the SHF is calculated as a function of $\overline{SLR}$. this is presented in Fig. 2.11.

The effect of varying $\overline{L}/\overline{L}_d$ (one manifestation of which could be the addition of night insulation i.e. $\overline{L}/\overline{L}_d$ decreases with the addition of night insulation); another of which could simply be the different climatic conditions; is shown in Fig. 2.11. As expected smaller values of $\overline{L}/\overline{L}_d$ (a larger fraction of heating load occurring during daytime), increases SHF. Load ratios of 2, 3 and 4 are considered.

In Fig. 2.12 the effect of varying the length of daytime period (dL) (with all other parameters fixed at their reference values) is presented. As (dL) decreases, SHF increaes. This is a consequence of the fact that the same amount of insulation is absorbed over a shorter daytime period (in a house with a time constant for heat transfer from storage to room air of 15 hr), and put to use at night when the load is greater.

Figs 2.15, displays the dependence of SHF on the relaxation time (Rt) (hrs) for several fixed values of $\overline{SLR}$. Increasing (Rt) corresponds to either increasing the storage thermal mass or decreasing the heat transfer coefficient from storage to room air or both (at fixed F). see eqns (1.3.11).

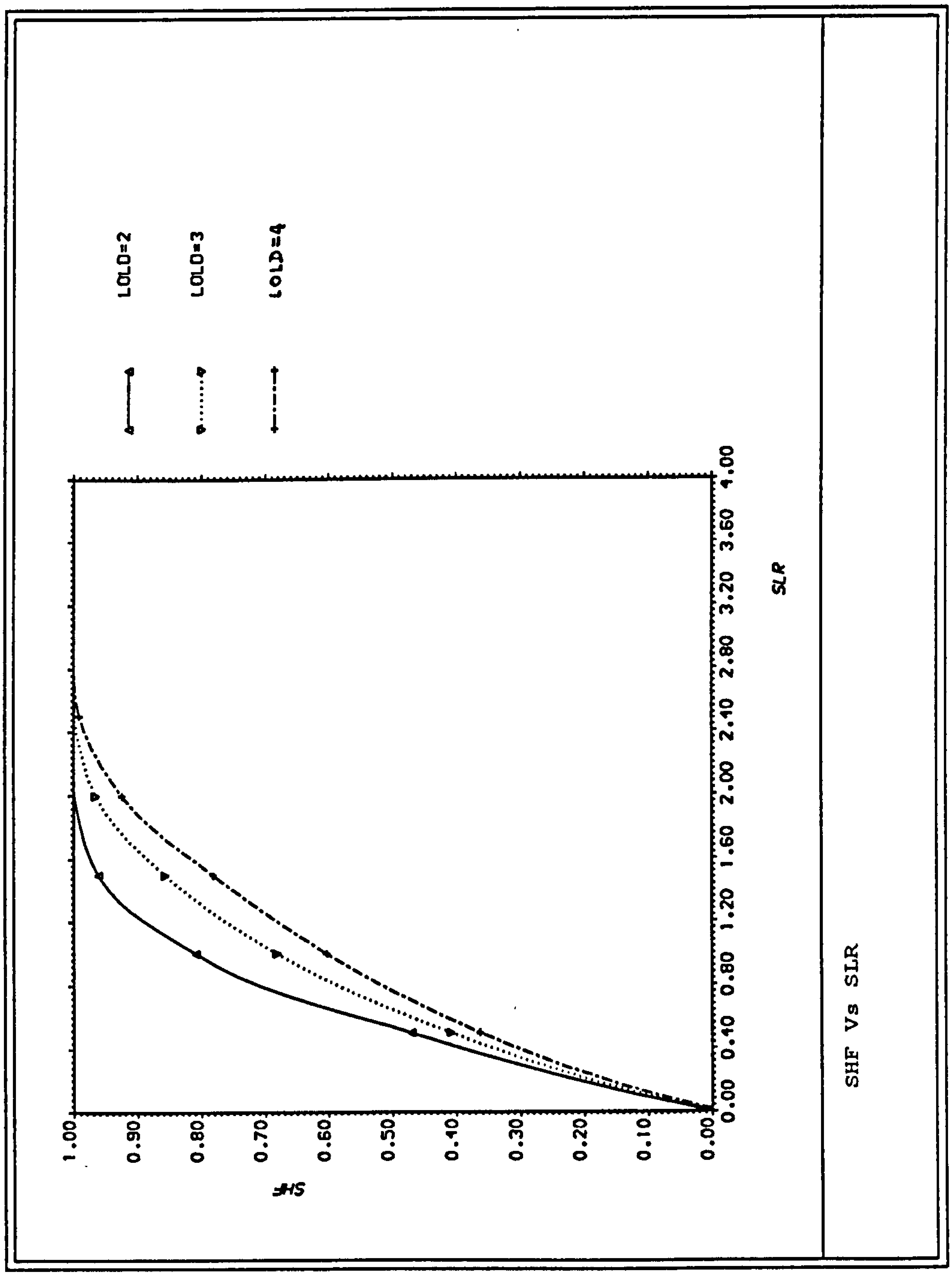

LOLD=2
LOLD=3
LOLD=4
SHF
SLR
1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10 0.00
0.00 0.40 0.80 1.20 1.60 2.00 2.40 2.80 3.20 3.60 4.00
SHF Vs SLR

Fig 2.11

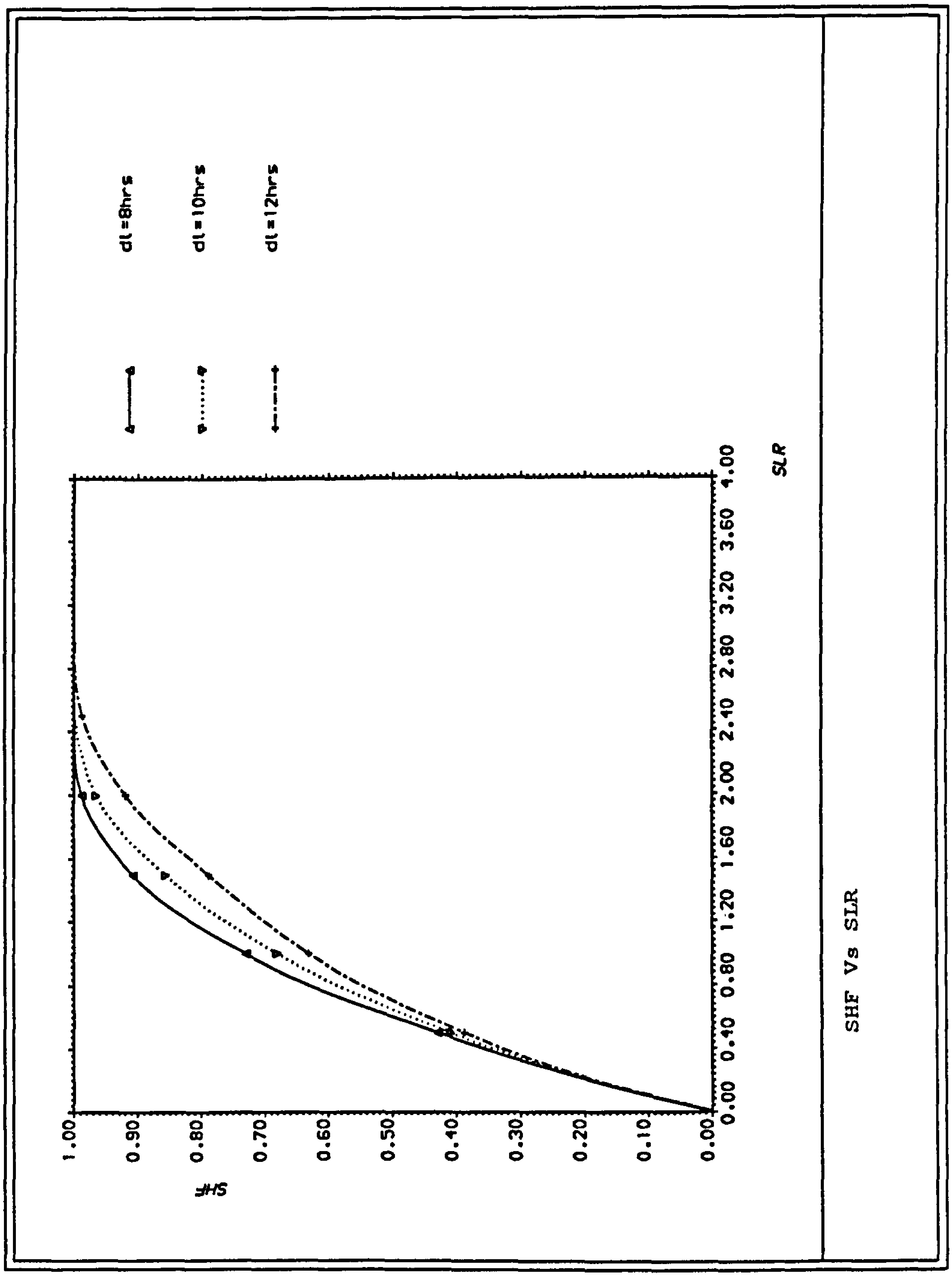
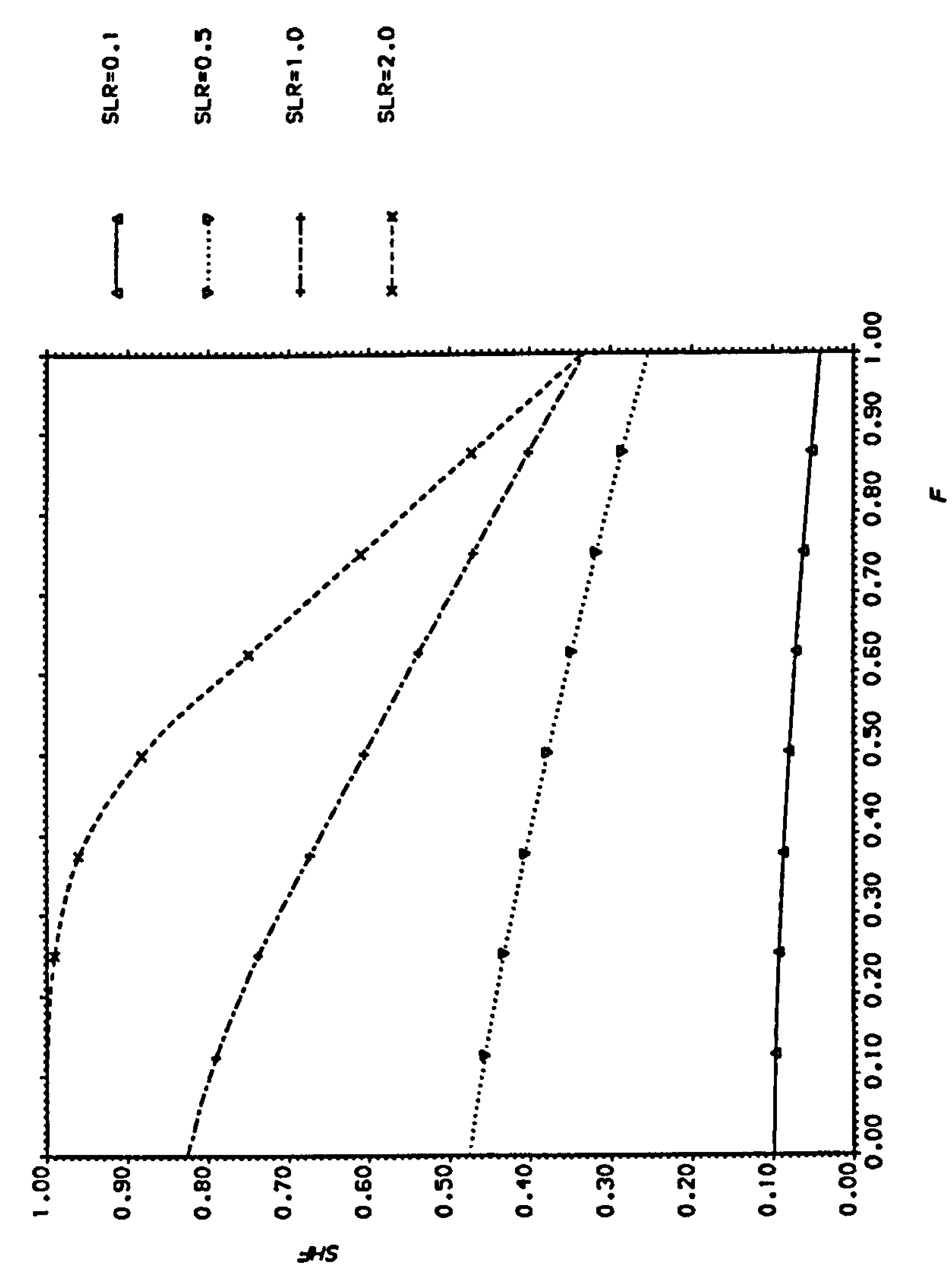

dl=8hrs
dl=10hrs
dl=12hrs
SHF
SLR
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.00 0.40 0.80 1.20 1.60 2.00 2.40 2.80 3.20 3.60 4.00
SHF Vs SLR

Fig 2.12



SHF Vs F

Fig 2.13

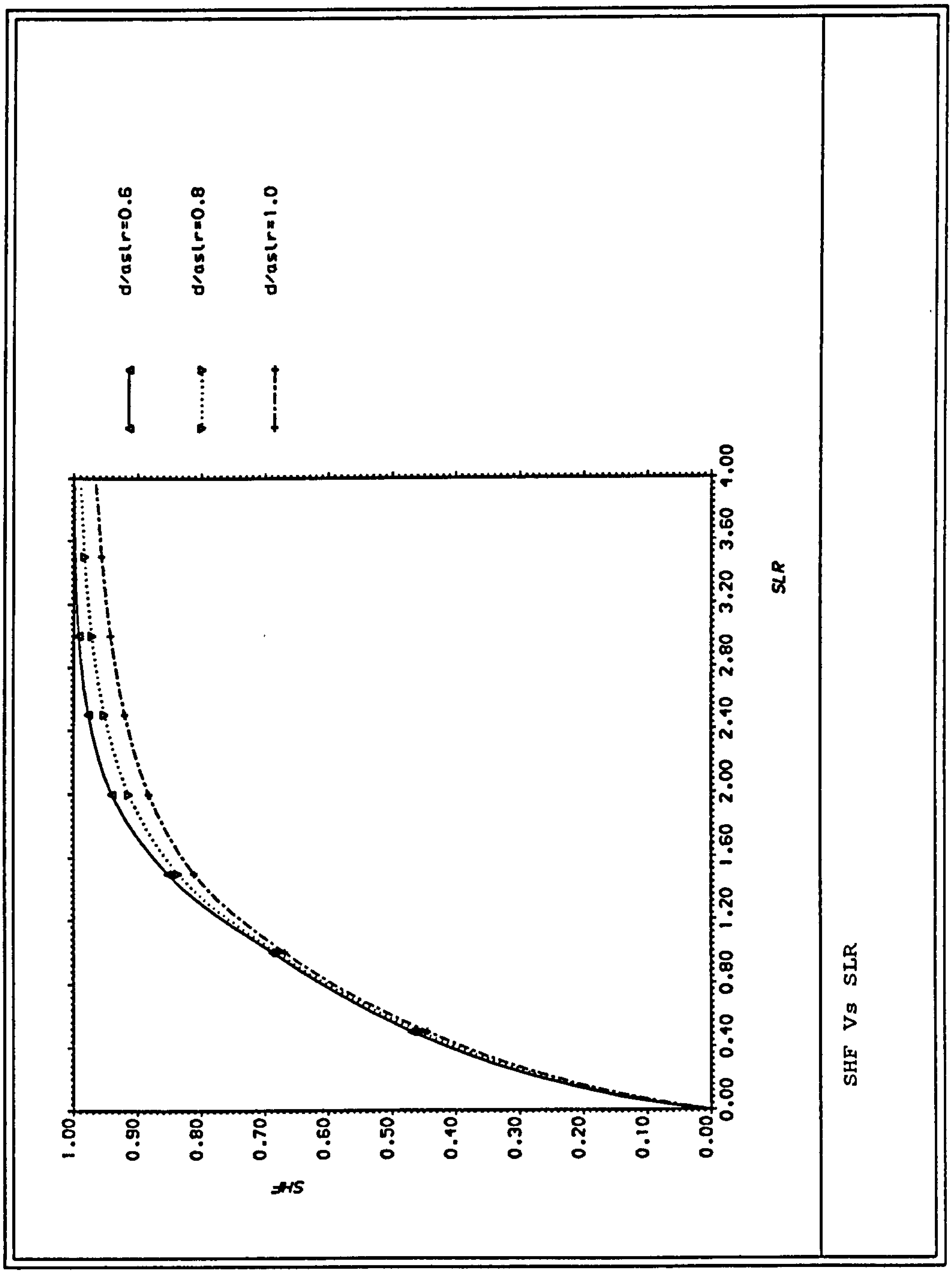
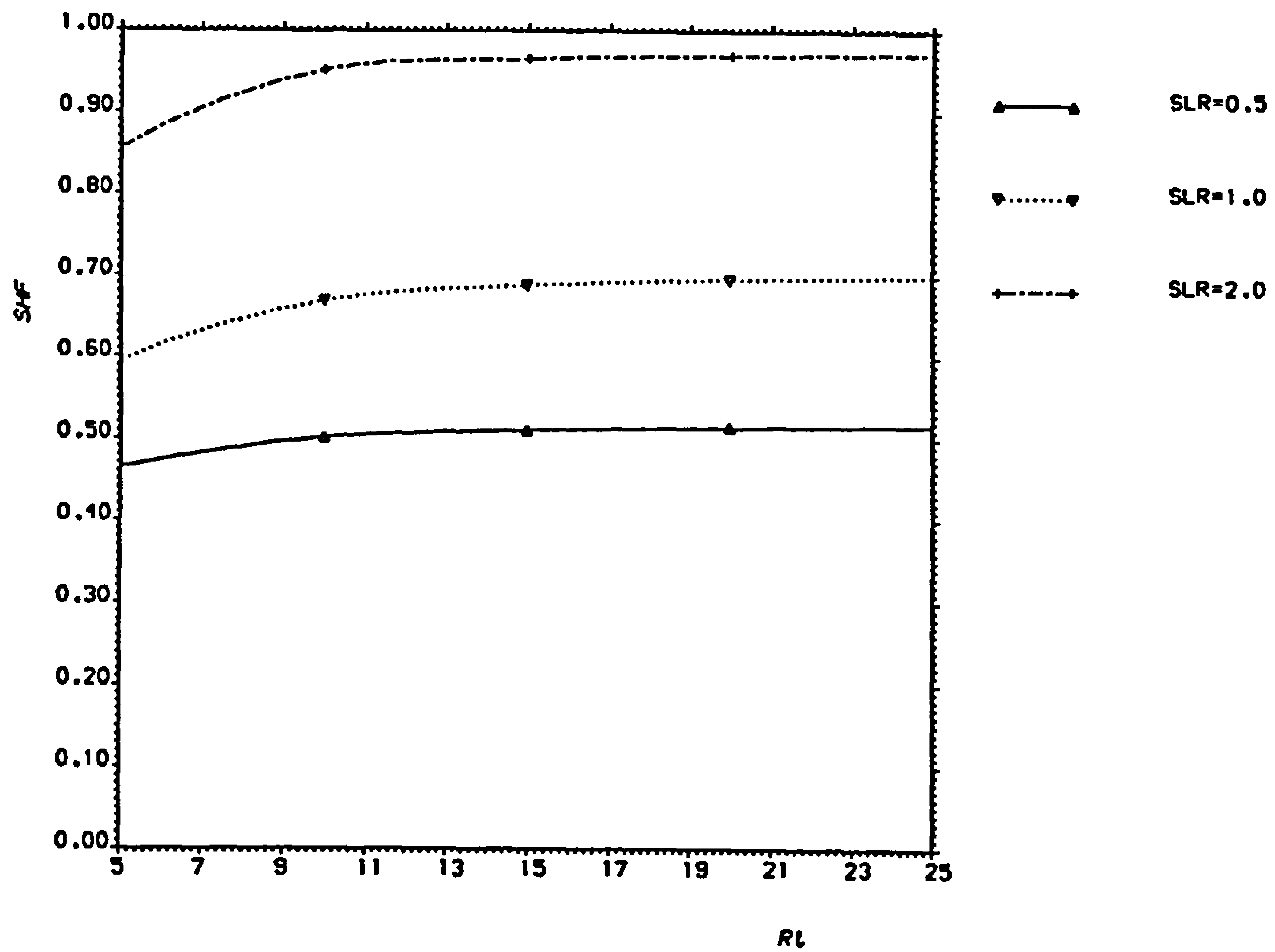

SHF
SLR
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
0.00
0.40
0.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00
d/aslr=0.6
d/aslr=0.8
d/aslr=1.0
SHF Vs SLR

Fig 2.14



SHF Vs Rt

Fig 2.15

It should be noted that increasing storage thermal mass by increasing its area rather than its thickness also affects the parameter F. SHF increases with increasing (Rt) and approaches an asymptotic value at large (Rt). These results are consistent with the findings of Ref. (4) and (1).

In Figs 2.13 the effect on SHF of varying F at fixed values of SLR is shown. At sufficiently low values of SLR, SHF is independent of F, and is about or equal to SLR.

At larger, more commonly encountered values of SLR, SHF is a decreasing function of F, which is in agreement with the findings of Ref. (24). This is due to the fact that Large F, results in larger fractions of daily insulation being absorbed directly by room air during daytime and less energy being stored for higher load night time periods.

We also note that at sufficently large values of SLR, the limiting value of SHF at F = 1.0 is simply L_d/L. This is a result of the fact that if no solar energy is being stored for night time use (F = 1.0), then sufficiently large SLR values will provide 100% of the daytime load, only.

Figs 2.14 shows the effect of varying the relative width of the parabolic distribution function, $\partial/\overline{SLR}$; shown in Figs. 1 and defined in eqns 1.3.1. It is noted that SHF is not a sensitive function of $\partial/\overline{SLR}$.

4.2 Study of the Conservatory

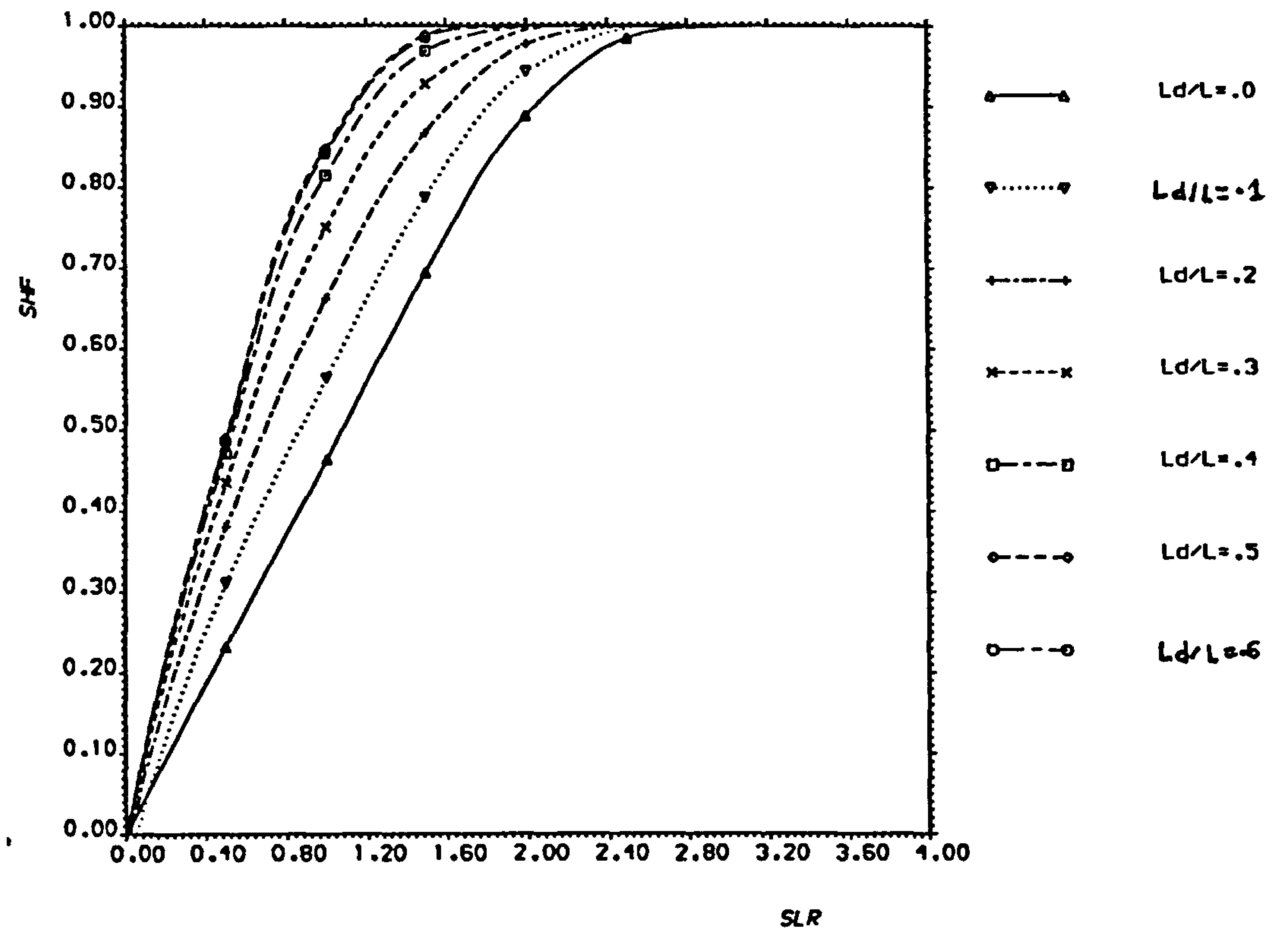
4.2.1 The direct gain, passive solar element with attached conservatory

Fig. (2.8) shows the flow diagram employed in studying the inclusion of a conservatory to the d.g. passive solar element. Again, as demonstrated in Figs 2.16, smaller values of $\bar{L}/\bar{L}_d$ or larger values of $\bar{L}_d/\bar{L}$ (larger values of heating loads occurring during daytime) increases SHF as for the simply d.g. house.

However, a fundamental difference in Fig.2.16 compared to 2.11 is the fact that for a given SLR & $\bar{L}_d/\bar{L}$, the d.g. system with attached solarium yields larger SHF values compared to the simple direct gain house. Consider ($\bar{L}_d/\bar{L} = 0.5$) in Figs. 2.11 and 2.16.

Figs 2.17 shows the effect of varying the length of daytime period (dL) on SHF for an element with attached solarium. Comparing figures (2.17) and (2.12) shows by the gradient of the curves (at fixed SLR and corresponding day lengths), that for a system with attached solarium the SHF values are generally higher.

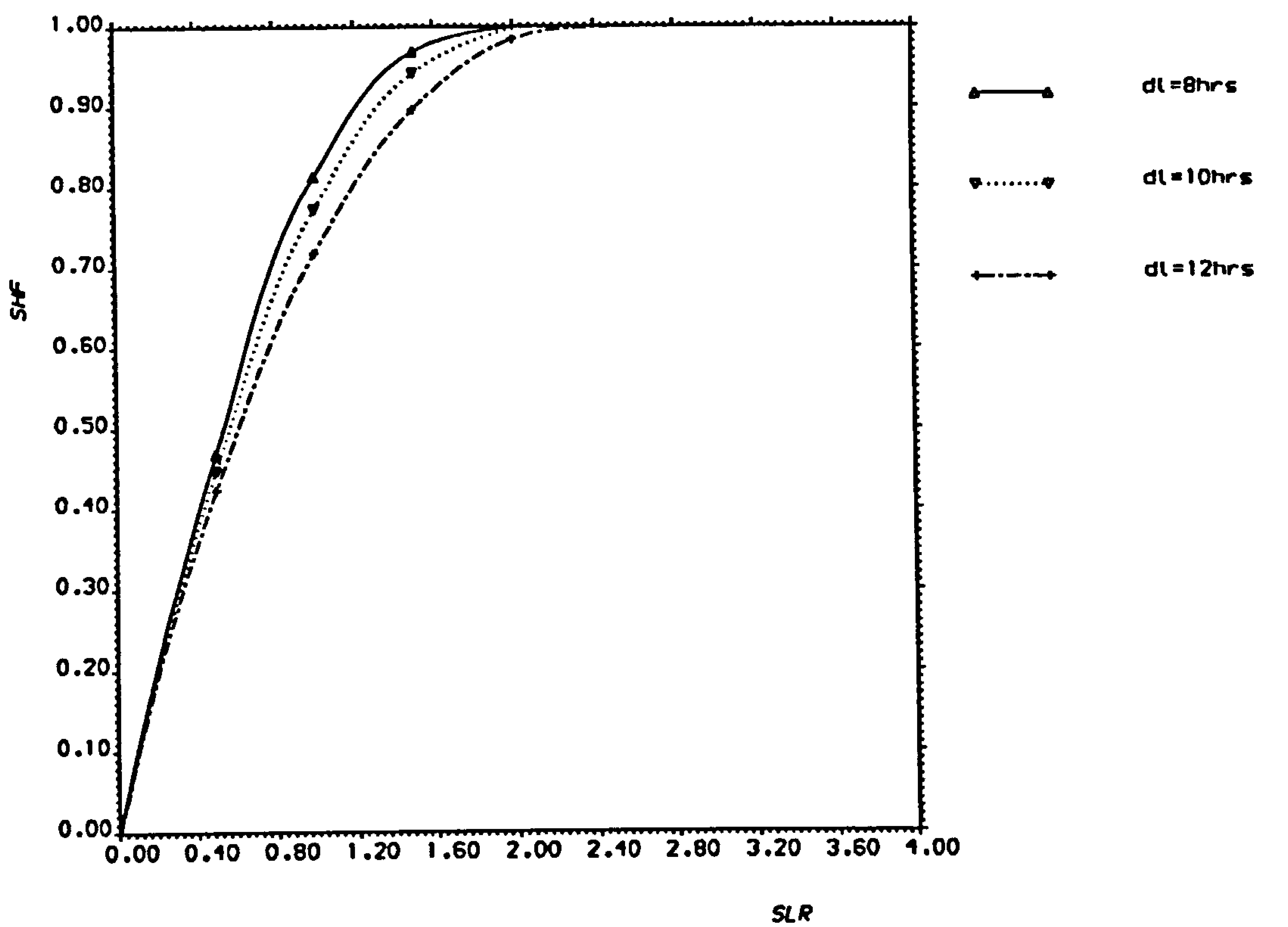
Figs. 2.18 shows a plot of the SHF vs (Rt) for fixed SLRs. As compared to Figs. 2.15 (the d.g. house without conservatory), the introduction of conservatory generates higher SHF values.



SHF Vs SLR (Dirrect gain system CONSERVATORY night insolation)

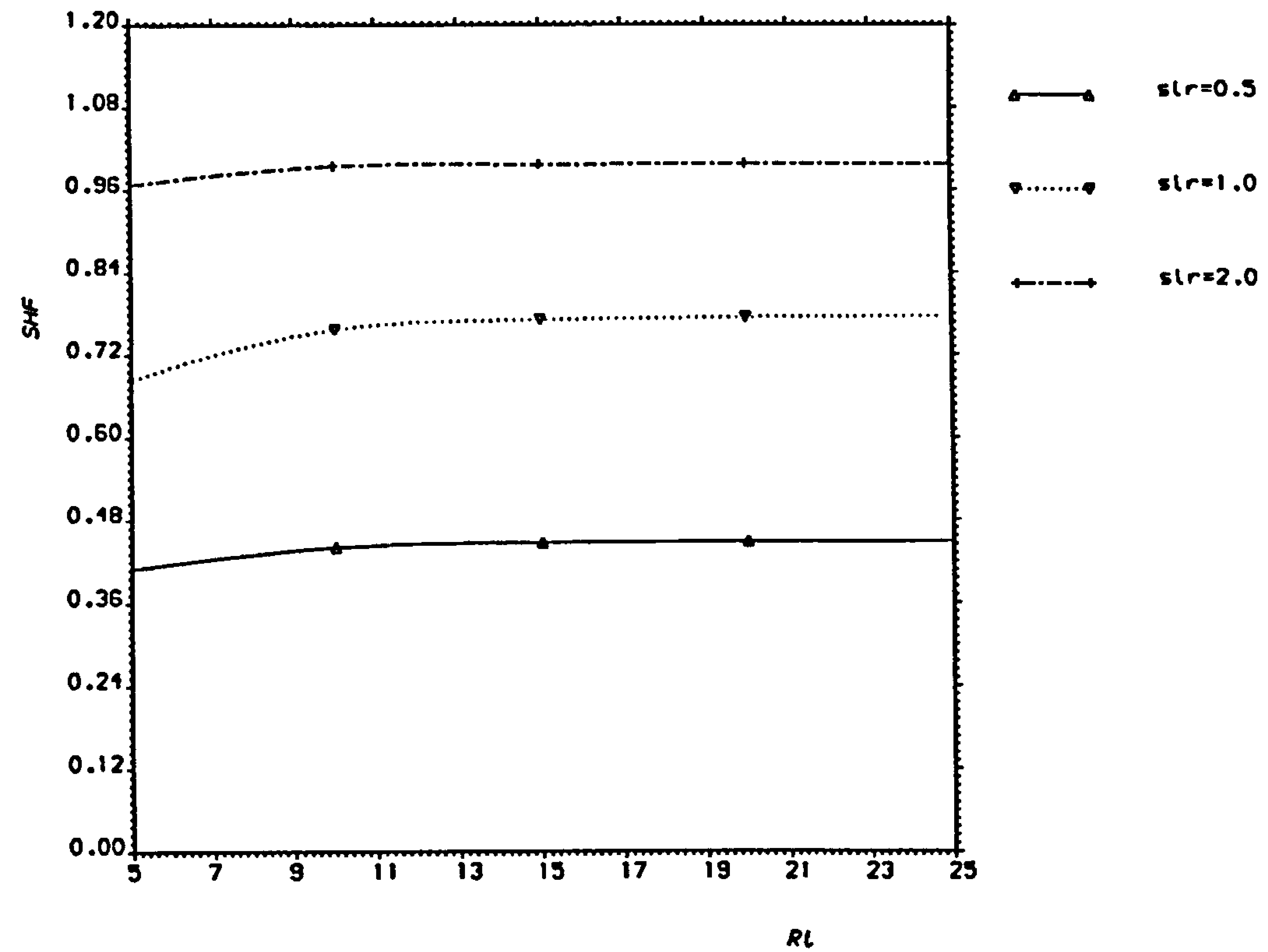
Tco(289k) 17'c B=.2 infil. rate Mi=9.4*10(-4)kg/sm2

Fig 2.16



SHF Vs. SLR d.gain with conservatory a fouth the size of house. M' By infilt. = 9.5 E-04 kg/sm2 air

Fig 2.17



SHF Vs. Rt direct gain house with conservatory B=0.2
Rt is the the daytime relaxation time

Fig 2.18

Figs. 2.20 shows that the introduction of conservatory for a fixed value of F_c does not considerably alter the variation of SHF with F. However, the variation of SHF with F_c (the amount of insulation through vertical windows not absorbed by conservatory storage) is noticeably different from the plot of SHF Vs F. F_c does not considerably vary with SHF for fixed SLRs. See Fig. 2.19.

Comparing figs.2.21 to figs 2.14 (the simply d.g. house) shows that the effect of varying the relative width of the parabolic distribution function $\partial/\overline{SLR}$ generally generates higher SHF values for the d.g. system with attached conservatory.

4.2.2 A study of the parameters affecting the building/conservatory geometry

U_{eq}, eqn. (2.2.5) was defined as the equivlent heat transfer coefficient from building envelop to ambient. As U_{eq} increases, it should be expected that the temperatures in the building envelop, hence conservatory should be much higher than ambient.

Figs. 2.27 shows that higher rates of heat transfer or coefficient occur for higher conservatory temperatures (above ambient: 11.6°C assumed).

Figs 2.28 illustrate the fact that for conservatory temperatures above ambient, the conservatory temperature increases with the conservatory size B with respect to the building.

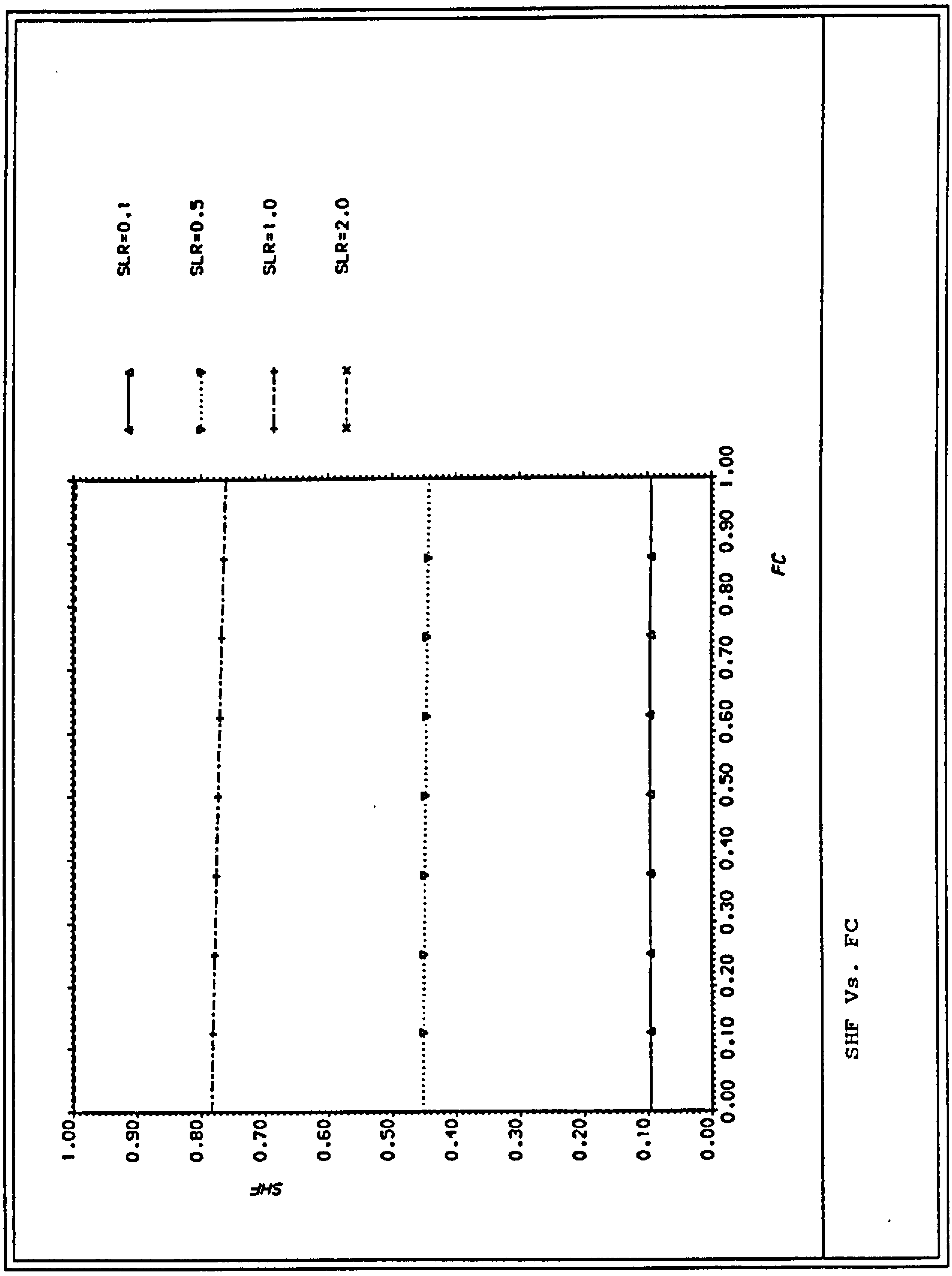
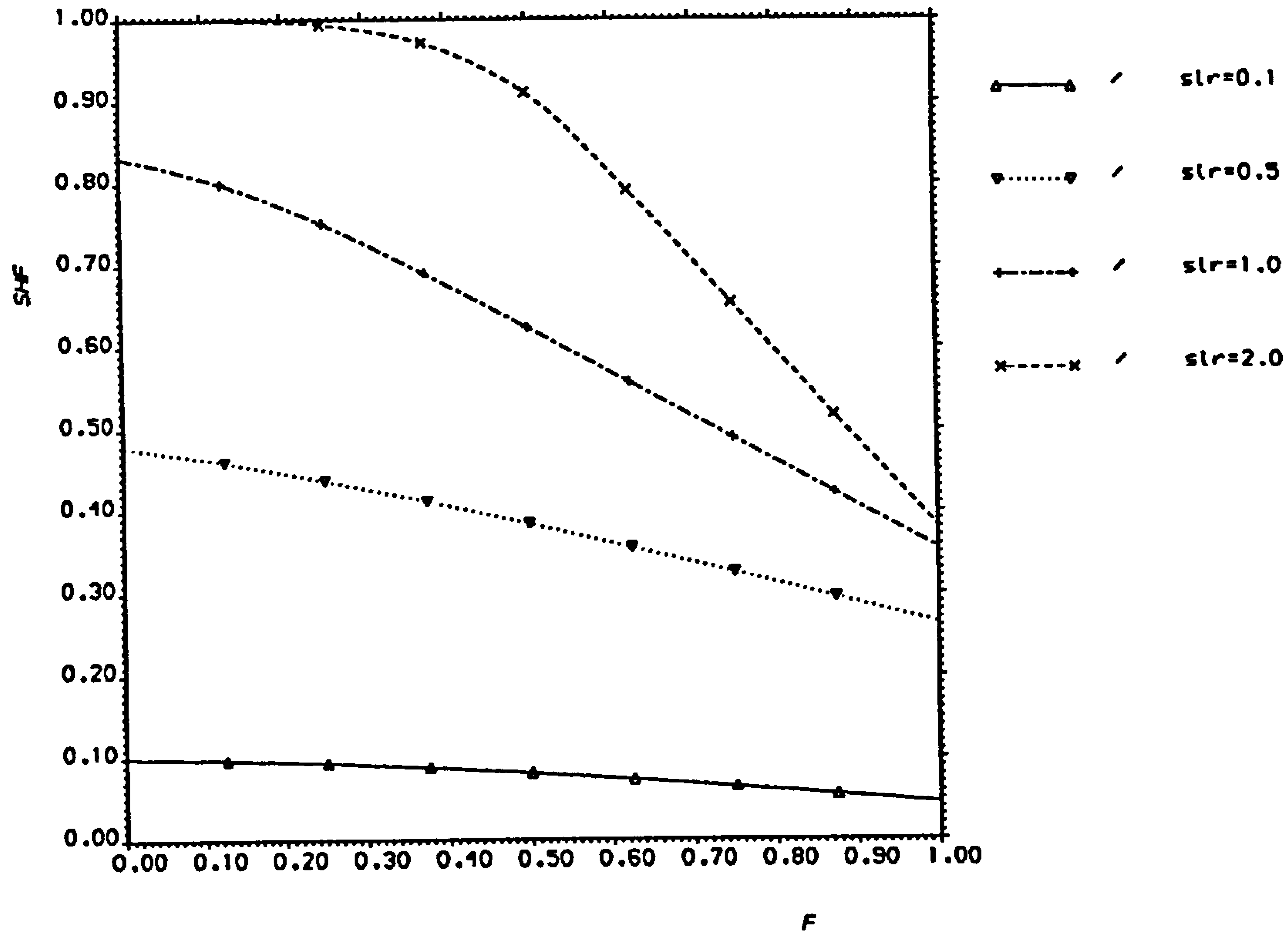

SLR=0.1
SLR=0.5
SLR=1.0
SLR=2.0
SHF
FC
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
SHF Vs. FC

Fig 2.19



SHF Vs. F (fraction of Qt not absorbed by storage)

D. gain house wtth conservatory. B=0.2

Fig 2.20

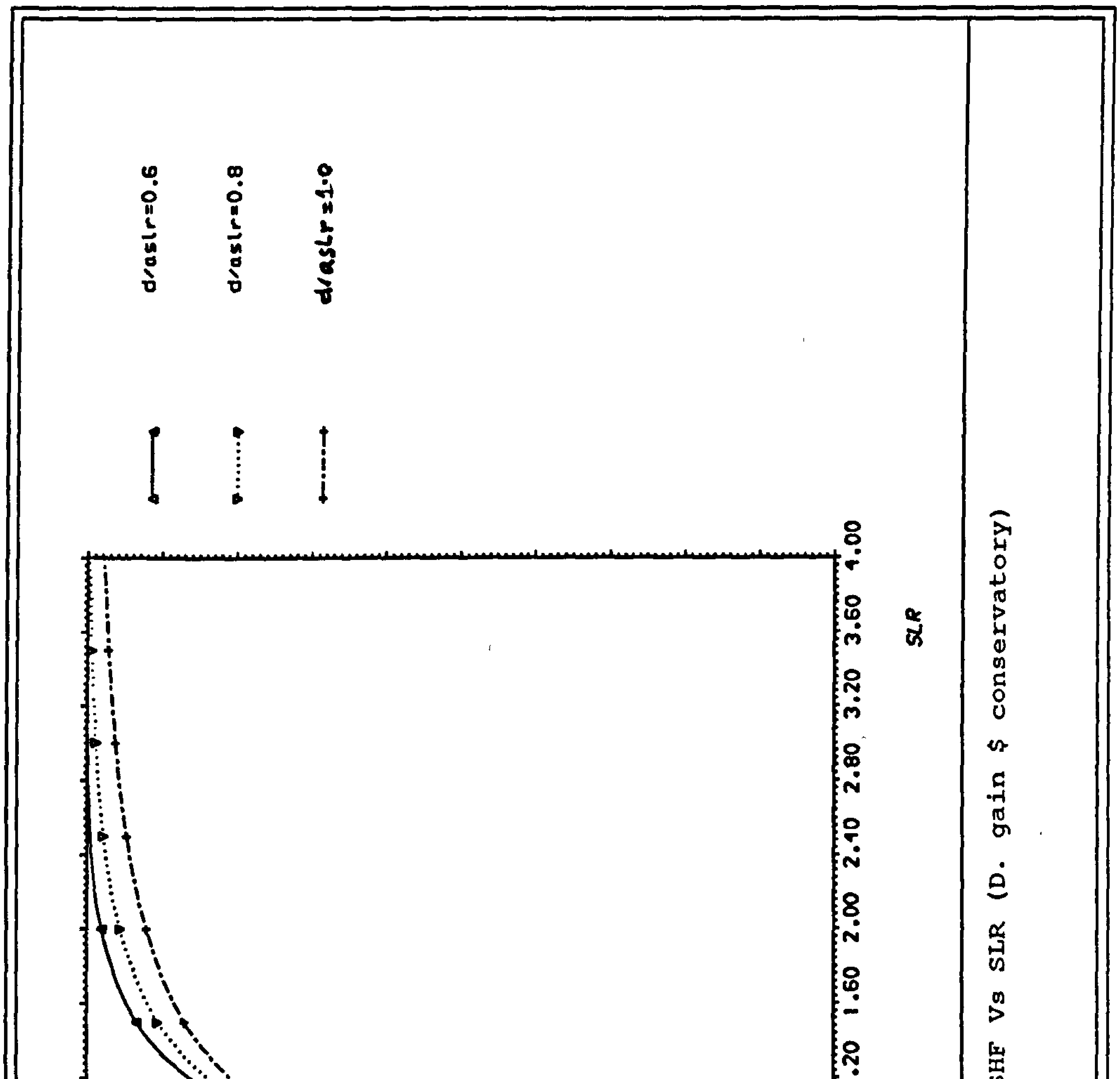

SHF Vs SLR (D. gain $ conservatory)
SHF
SLR
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.00
0.40
0.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00
d/aslr=0.6
d/aslr=0.8
d/aslr=1.0

Fig 2.21

Another feature of Figs. 2.28 is the fact that the difference between a simply direct gain feature B=0 and a greenhouse B = 1.0 at same weather conditions is a temperature difference of about (22.1 - 15.6) or 6.5°C.

On considering eqn. 2.13 it was mentioned that the effect of the multiplier F_R (heat removal factor) is to reduce the calculated useful energy gain from what it would have been had the whole system been at ($T_{f,i} = T_{co}$); to what it actually is using a fluid that increases in temperature as it flows through the system. Thus higher values of building envelop temperature from actual flow values or larger U_{eq} values should show a decrease in the multiplier F_R from actual. This is seen in Figs. 2.29 , where for higher U_{eq} values, F_R decreases.

Again, from the definition of the multiplier F_R above (heat removal factor), and as already pointed out higher conservatory sizes (B - values), approaching the greenhouse (B = 1), represent higher conservatory temperatures; as the actual building envelop temperature increasing differs from conservatory temperature the multiplying factor F_R (heat removal factor) decreases. This is illustrated in Fig. 2.30.

Figs. 2.22 and 2.23 illusttrate the variation of the multiplying factor F_R (heat removal factor); with $\dot{m}C_p/U_{eq}$, for various conservatory sizes. Firstly, the size of the conservatory does not considerably affect this pattern.

More importantly as the storage mass increases or heat transfer coefficient U_{eq}, from building envelop decreases, the heat removal factor asymtotically approaches its actual value (i.e. unity).

This is due to the fact that U_{eq} the heat transfer coefficient from building envelop to ambient has decreased considerably and more uniform conditions in the building envelop can be more represented by a single temperature e.g. T_{co} .

Figs 2.24 shows that for low ventilation or infiltration rates the temperature difference for various conservatory sizes is much more marked than at higher ventilation or infiltration rates. Also for any given ventilation or infiltration rate, the conservatory temperature asymtotically reaches ∞ for no ventilation or infiltration. $\dot{M}_v$, $\dot{m}_I$ = 0.0. Also, for any given ventilation or infiltration rate larger conservatories generally represent higher temperatures.

Figs. 2.25 again illustrates the fact that the multiplier F_R decreases as conservatory temperature T_{co} increases for any conservatory size. At $T_{co} = T_a = 11.60\,^\circ C$, the heat removal factor is unity as should be expected from the definition of F_R .

Larger conservatories (B - values) illustrate a more marked variation in F_R versus T_{co} . This is because the actual difference in building envelop temperature from conservatory temperature is much more pronounced with the introduction of larger conservatory.

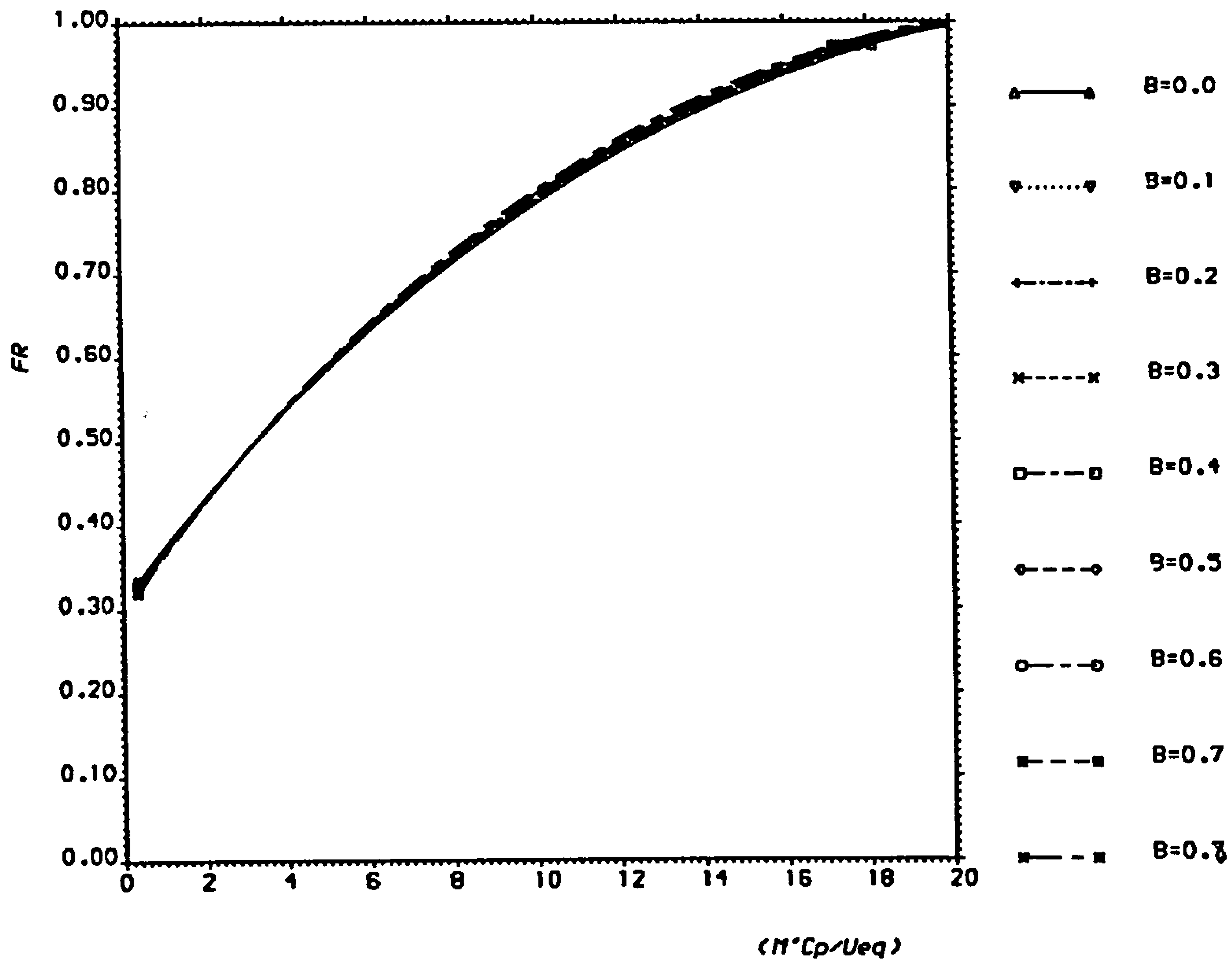
Figs. 2.26 illustrate that the equivalent heat transfer coefficient from building envelop to ambient, U_{eq}, increases with increasing conservatory size w.r.t building (B - value).

Figs. 2.30 shows that the heat removal factor or multiplier F_R decreases for larger conservatories as should be expected (from a simple d.g. B = 0, to a greenhouse B = 1).

Figs 2.31 illustrate that the T_{co} decreases with increasing infiltration rates into conservatory at all insolation levels considered.

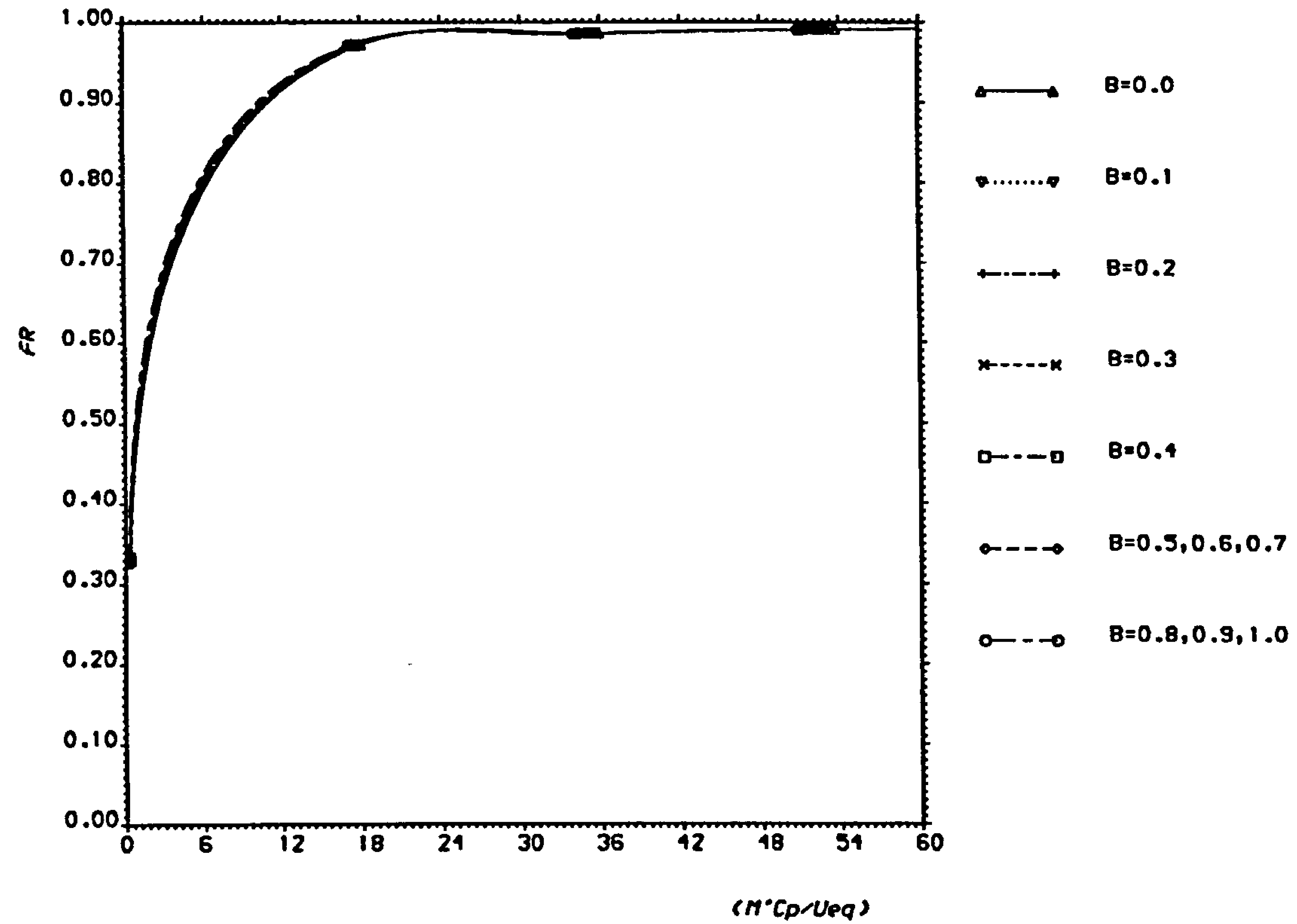
Figs 2.32 and 2.33 illustrate the frequency of occurrence of SLR values versus specific SLR values using Typical Reference Year (TRY) hourly data for Milton Keynes U.K. Lat (52.8N) (Linford passive solar houses); and Kew, London (10 yrs reference).

A 5th Degree polynomial proved sufficiently accurate to represent both models; with a multiple correlation coefficient of R = 0.97 and R = 0.98 respectively. The results thus validate the approach of Cowing and Kreider (20) validating them for U.K. locations.



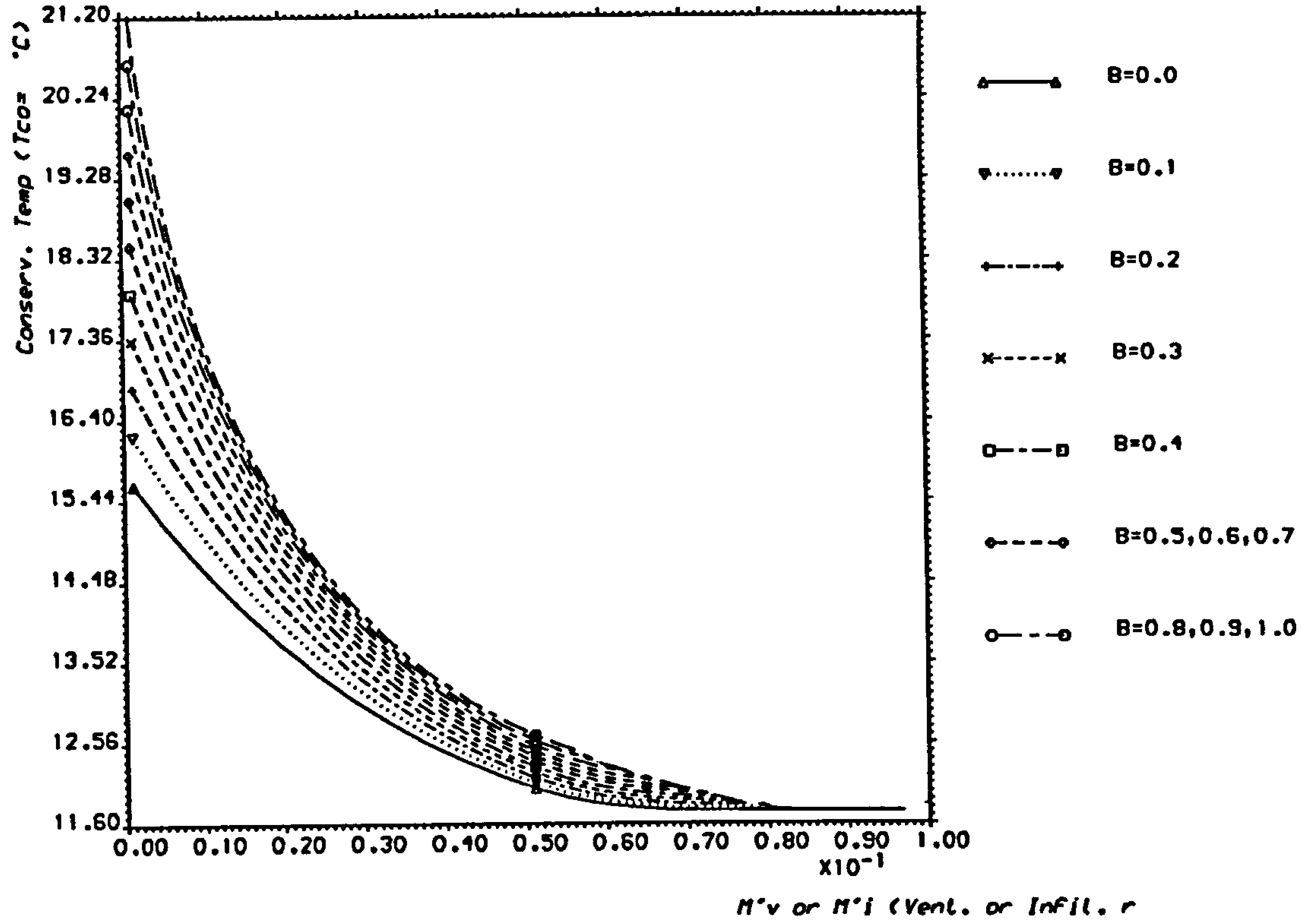
Heat removal factor FR Vs. (M'Cp/Ueq)

Fig 2.22



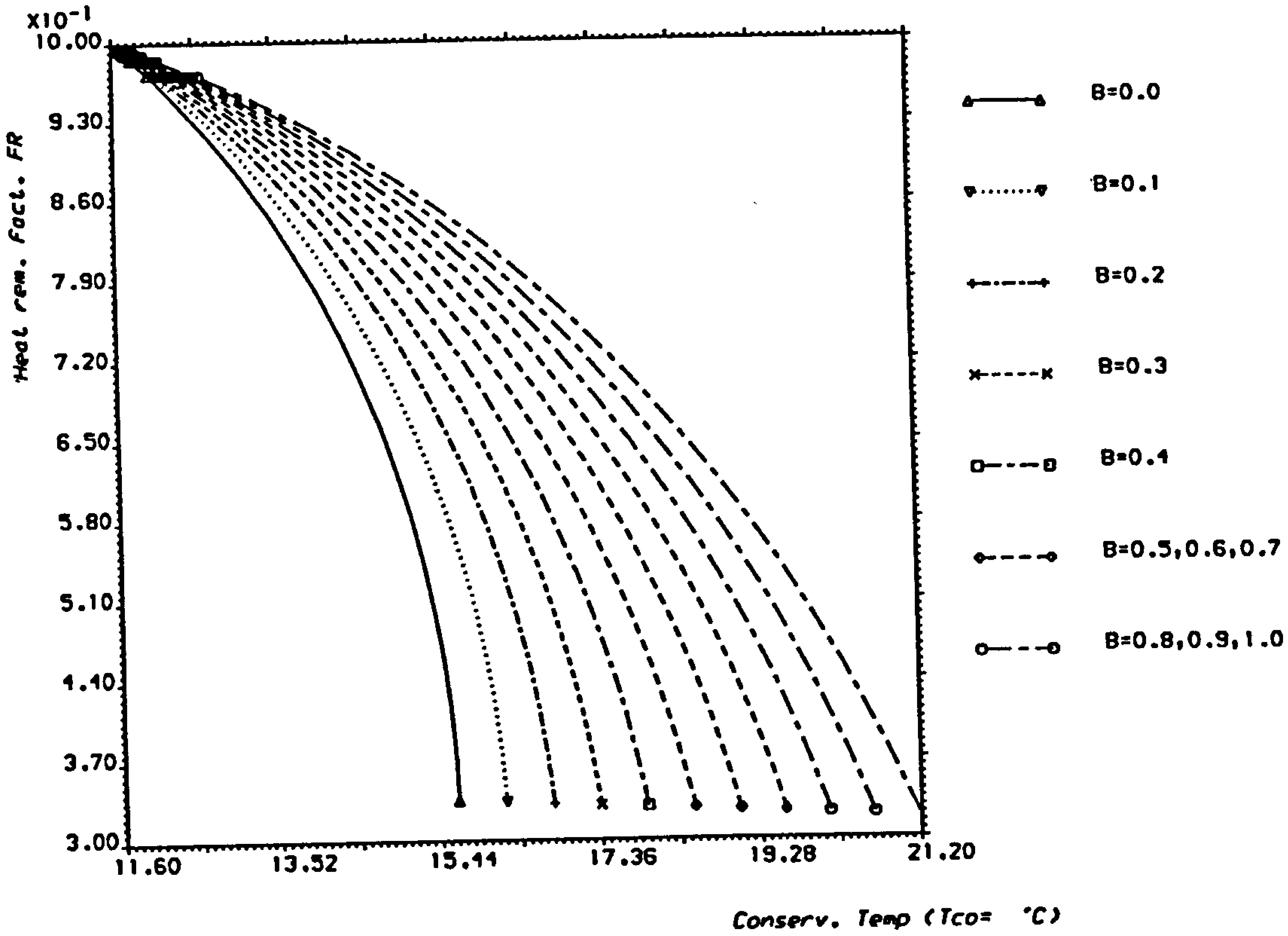
Heat removal factor FR Vs. (M'Cp/Ueq)

Fig 2.23



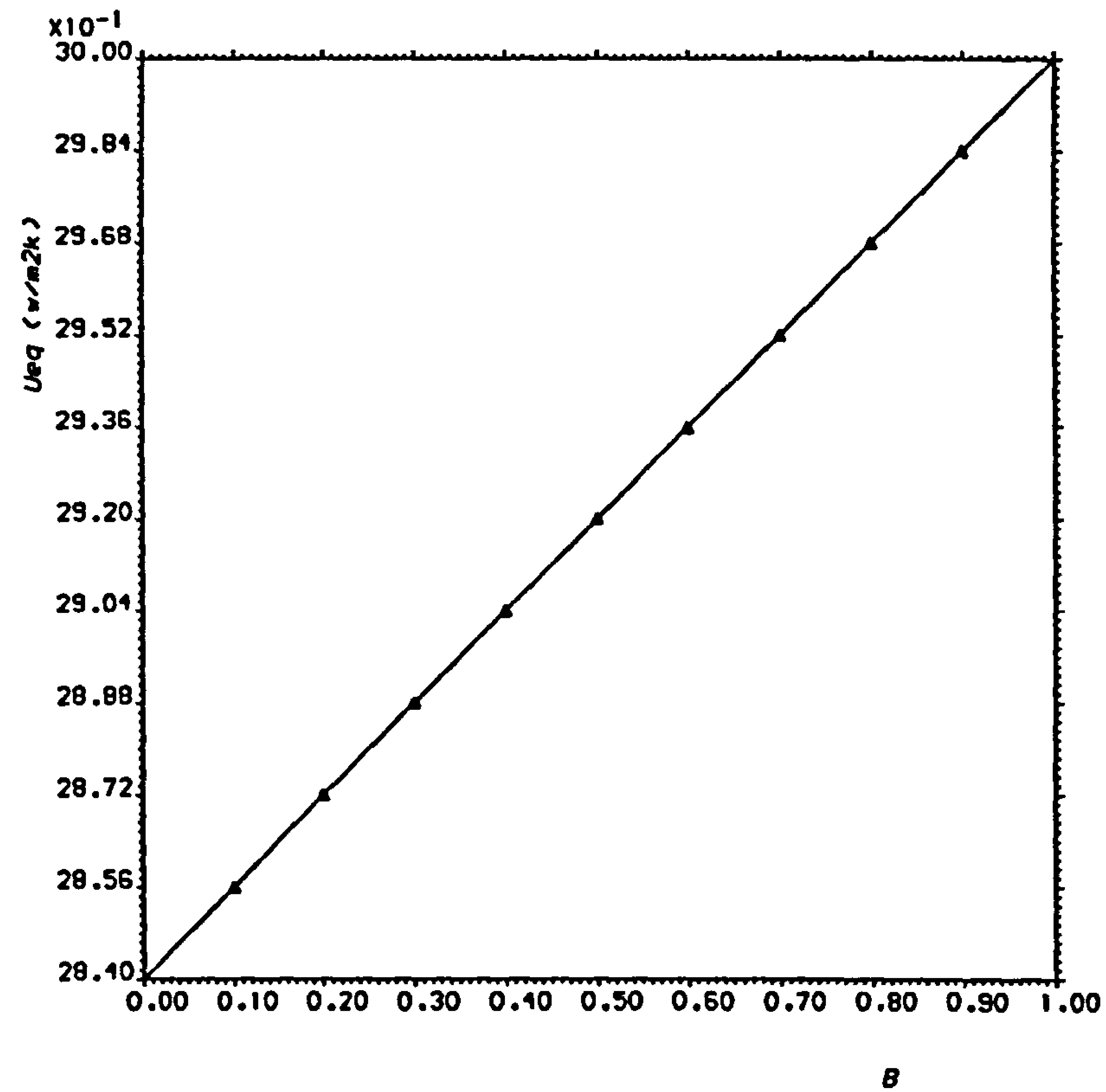
Conserv. Temp Vs. Mass flow rate into Building envelop.

Fig 2.24



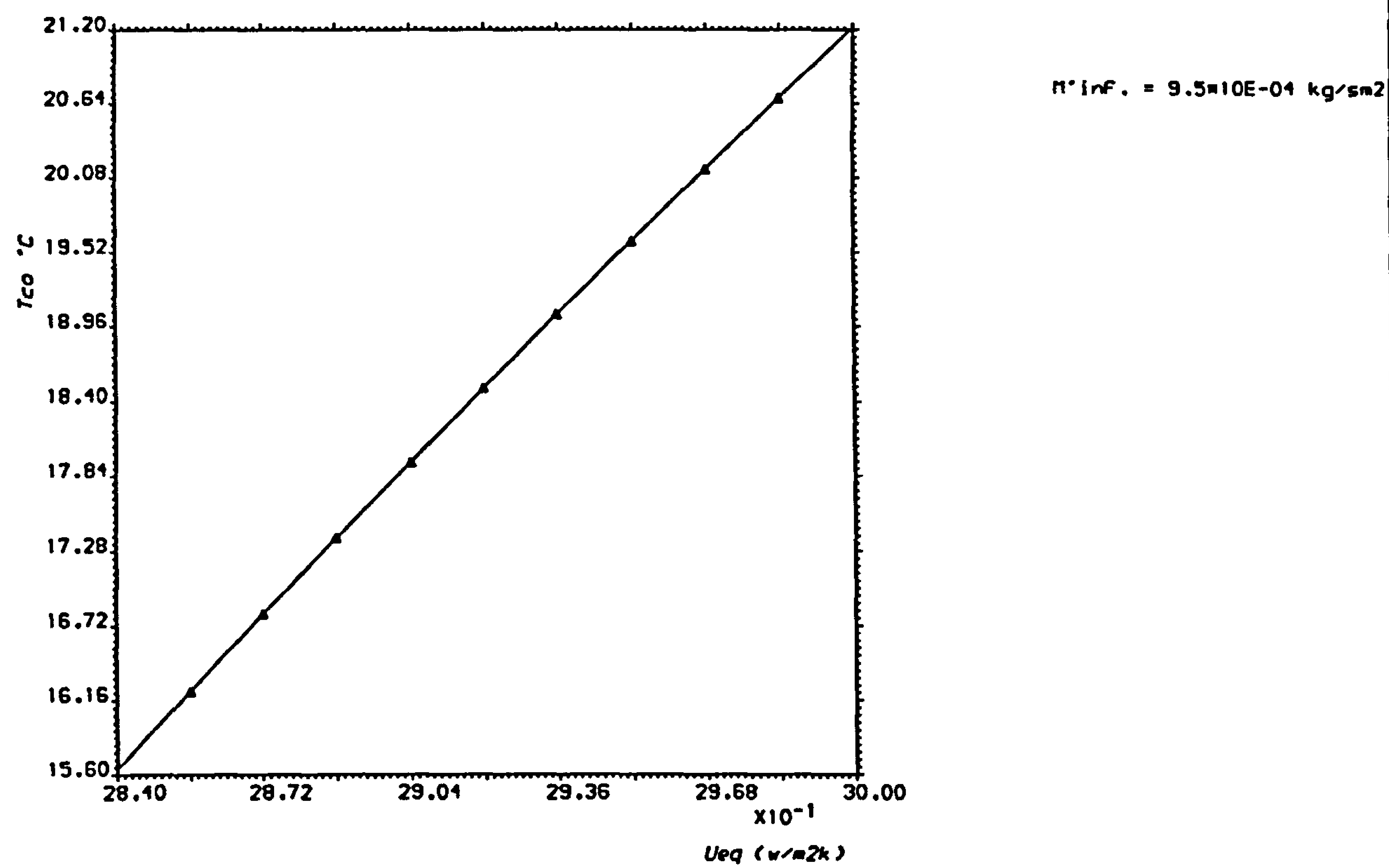
FR of Building envelop Vs. Conserv. Temp

Fig 2.25



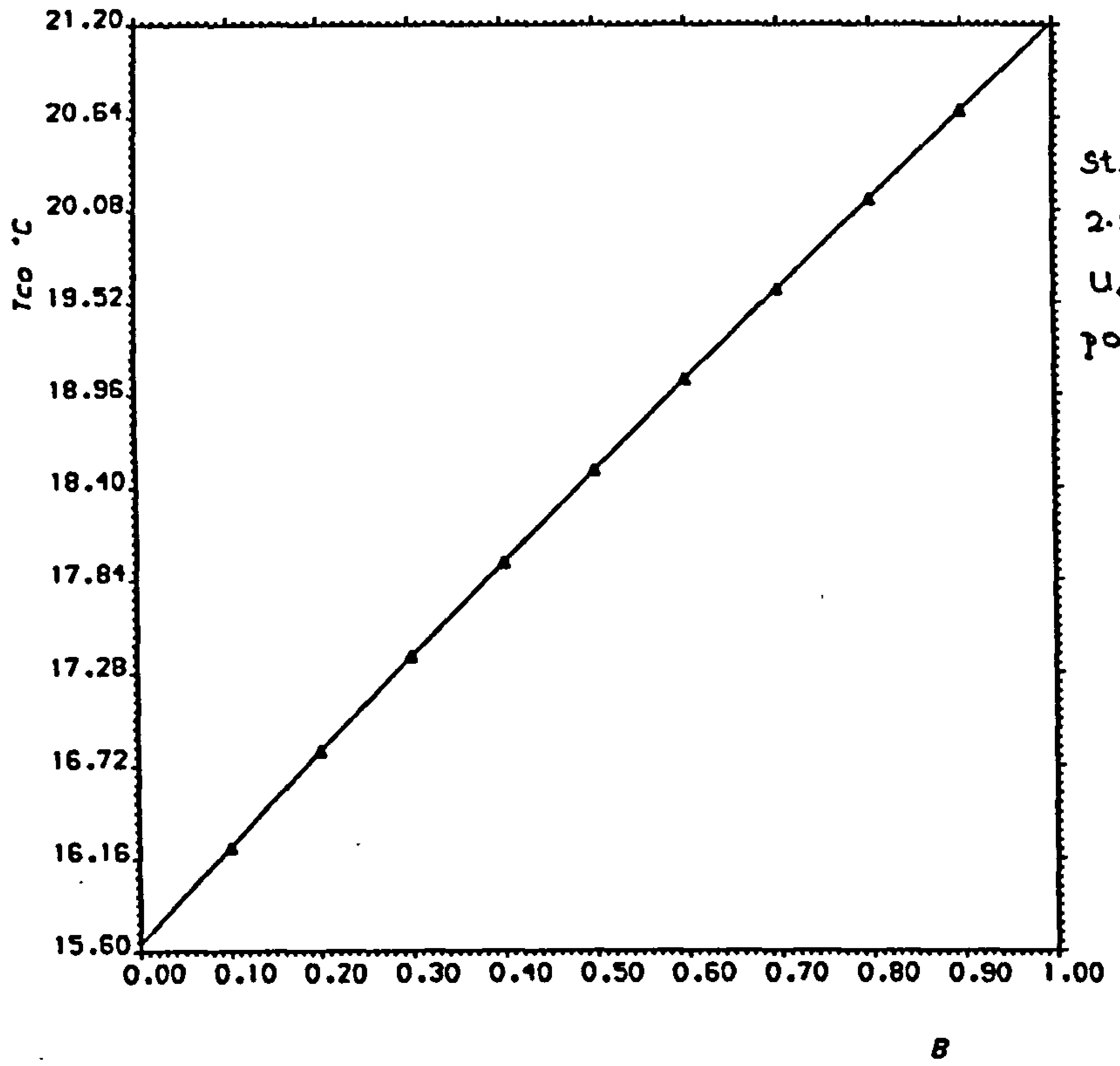
Plot ,Equivalent heat trans. coeff from building evvelop to amb.(Ueq),Vs.Volumetic prop. from unity of air in conserv.B

Fig 2.26



Conserv. Temp. for increasing heat trans. coeff. from envelop of building to ambient .

Fig 2.27

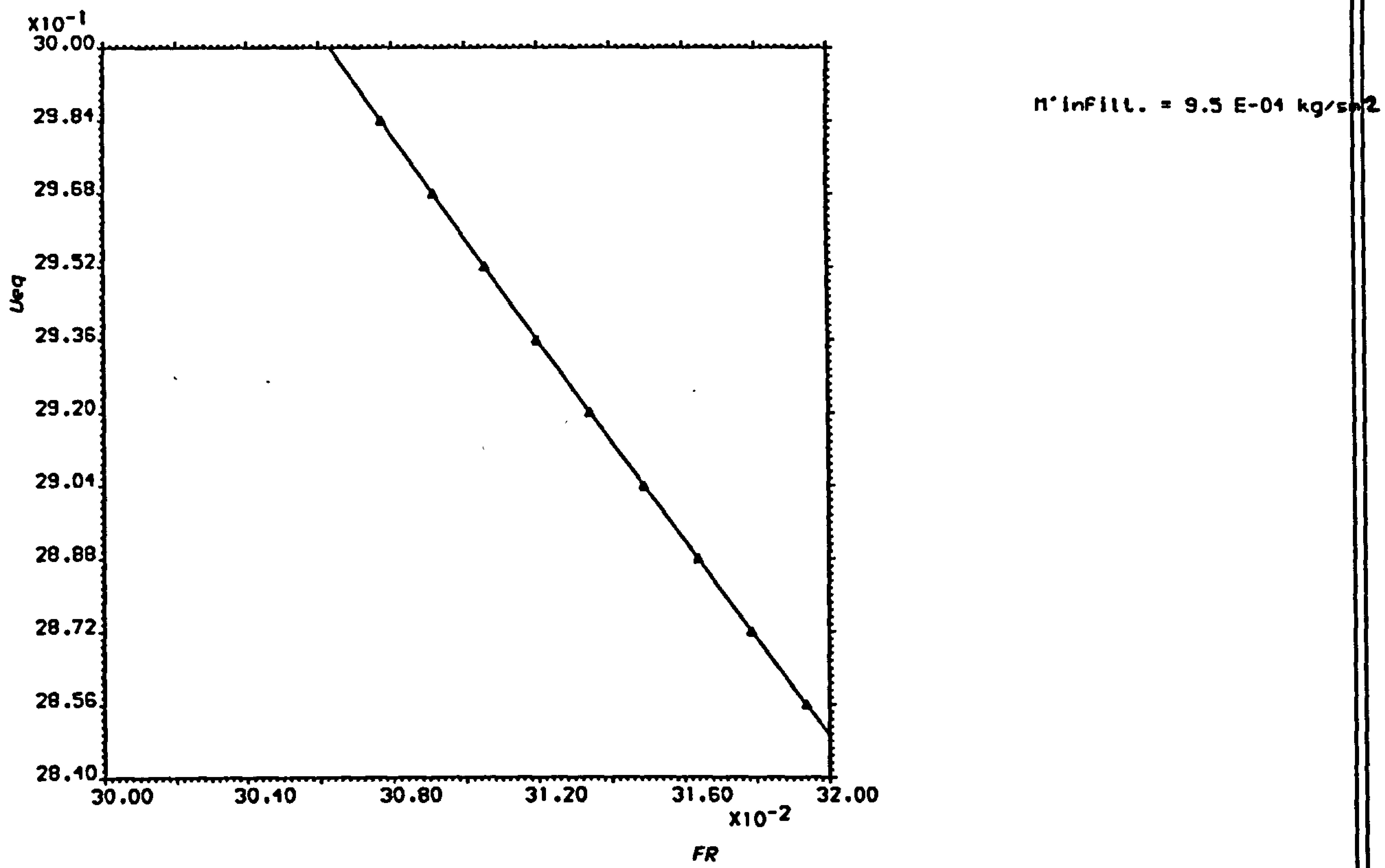


M'infil = 9.5E-04 kg/sm2

Strickly speaking eqn 2.2.5 and 2.17 shows varying B varies U_{eq} as well as T_{co}. At this point U_{eq} is not varied.

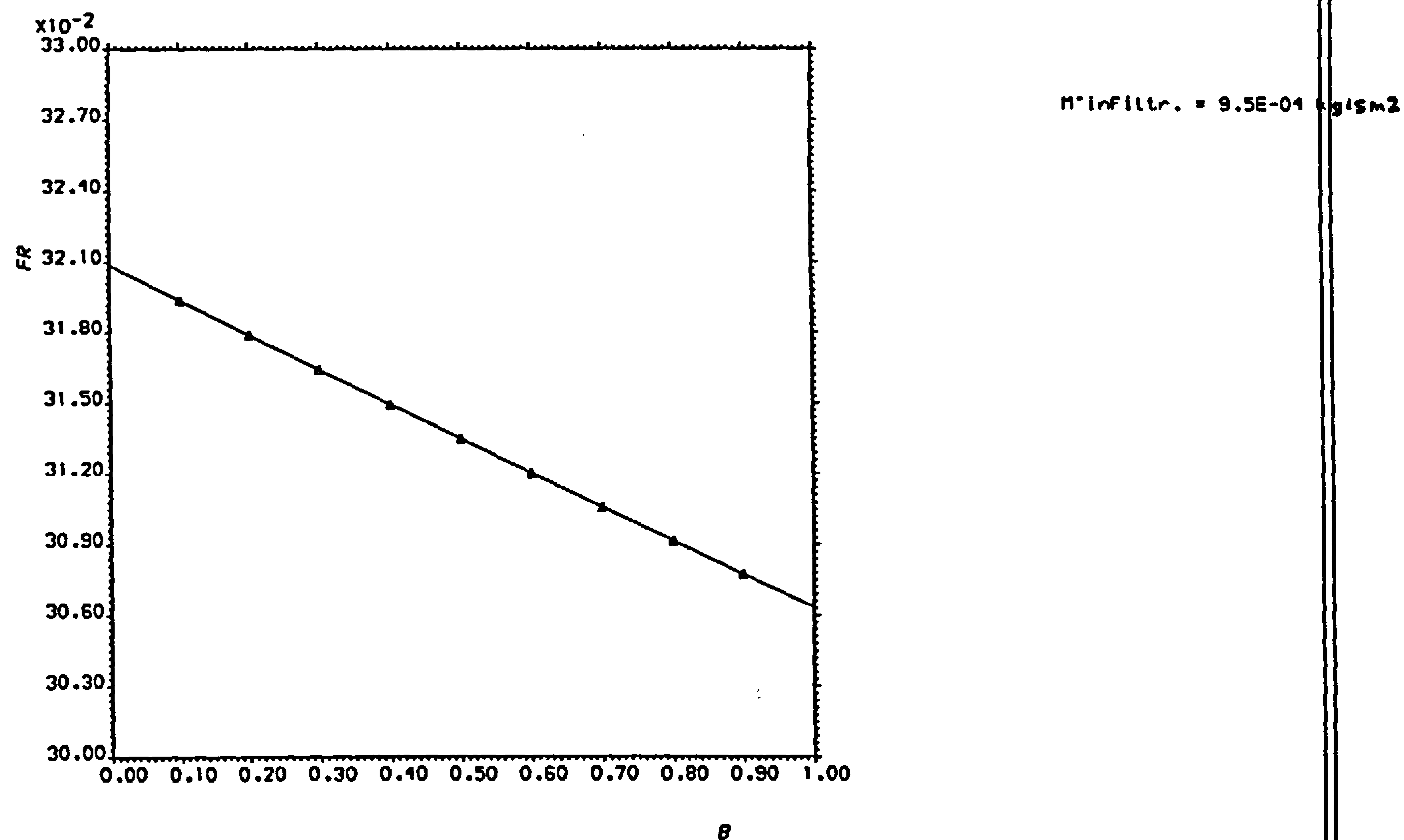
Conserv. temp 'C for ascending cons. size (B),(w.r.t the building and unity.)

Fig 2.28



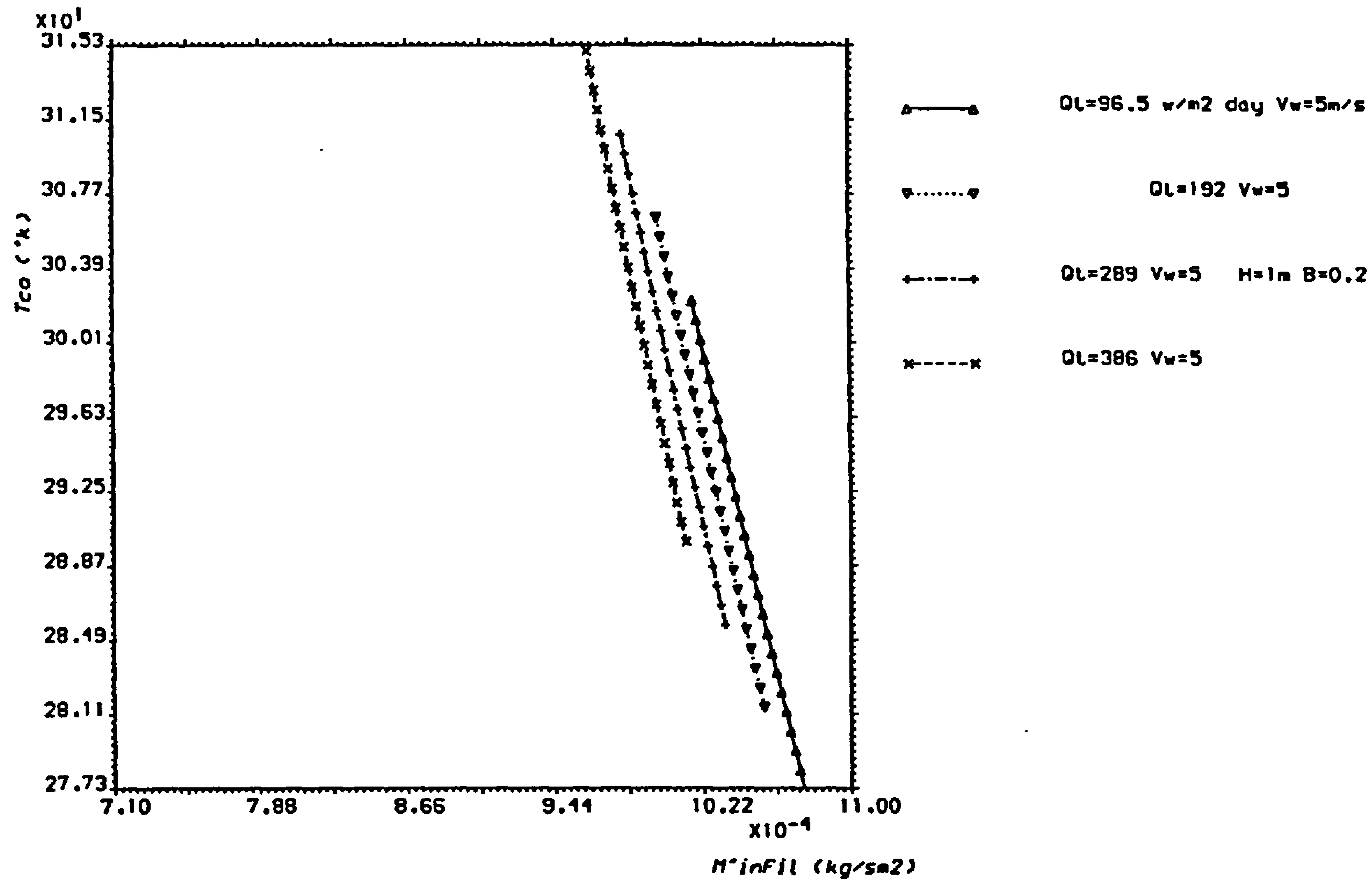
Effect of increasing heat trans. coeff for building envelop
Ueq on heat removal factor FR.

Fig 2.29



Effect of larger conservatories on heat removal factor. (flow by infiltration)

Fig 2.30



The conservatory temperature as a function of the infil.
rate into the conservatory,at various insolation levels.

Fig 2.31

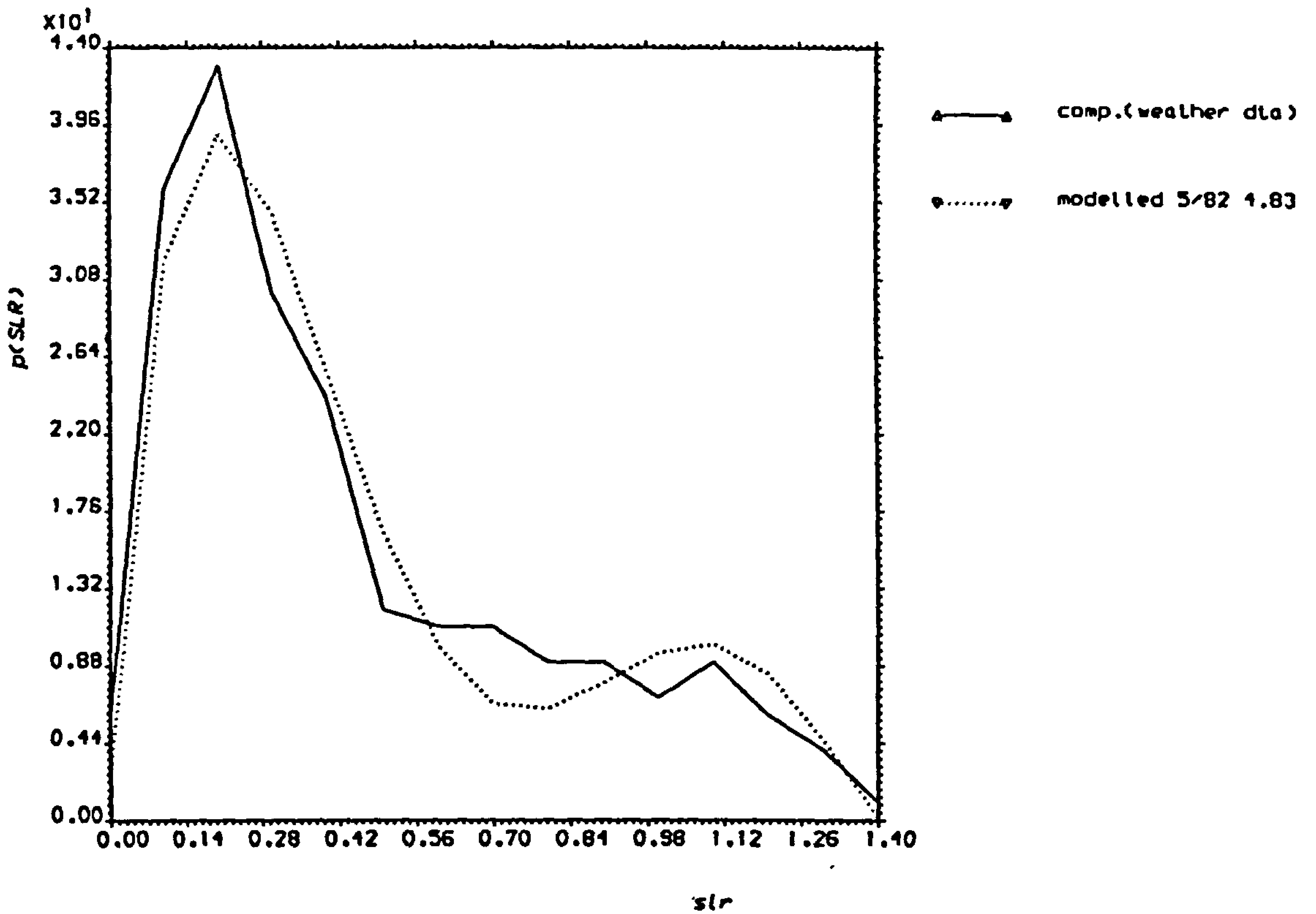
5th Degree Polynomial

$$p(SLR) = Ag + Bg\,SLR + Cg\,SLR^2 + Dg\,SLR^3 + Eg\,SLR^4 + Fg\,SLR^5$$

MILTON KEYNES LINFORD HOUSES

R = 0.971623

A = 2.787217
B = 441.568843
C = -1750.04674
D = 2552.68965
E = - 1600.924
F = 363.454547



P{slr} as a function of slr. Linford passive solar houses, Milton Keynes 52.8N

Fig 2.32

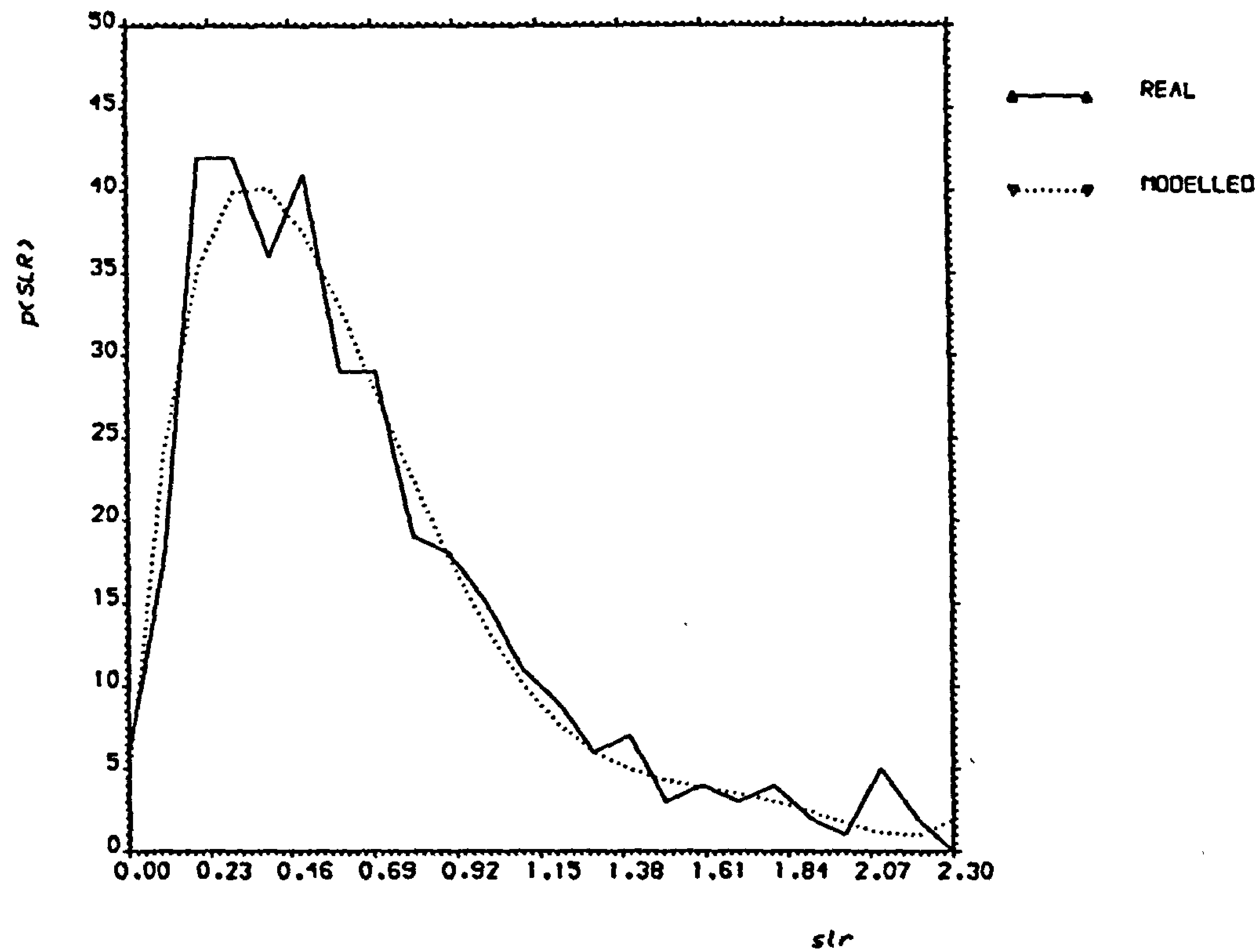
5th Degree Polynomial $p(SLR) = Ag + Bg\,SLR + Cg\,SLR^2 + Dg\,SLR^3 + E_g SLR^4 + F_g SLR^5$

KEW GARDENS, LONDON

R = 0.979757

* Unnormalised cfts

A = 4.788941
B = 246.087936
C = -555.087936
D = 469.135382
E = -175.80728
F = 24.55326



The freq. of occurrence of SLR values Vs. specific slr using TRY hourly data for kew, london (10 yrs ref)

Fig 2.33

It was then useful to apply both the analytical techniques employing a parabolic p-SLR and polynomial p-SLR, detailed in sections 1 and 2 of this paper, to actual monitored building and weather data.

Data available for the Passive Solar Heated Linford Houses in Milton Keynes for the Heating Season May 82-April 83 was used and the SHF,SLR distribution for 3 houses (33,35 and 36) obtained.

Both approaches were applied to each of the 3 individual houses for different load ratios $(\bar{L}d/\bar{L})$ and all 3 houses together using both methods. The SHF,SLR distribution for 3 daytime to total load ratios $(\bar{L}d/\bar{L})$ for Houses 33,35 and 36 are presented in Figs.(2.34) (2.36) and (2.38) respectively using method 1.

For the house load ratios considered the weather data showed that both houses 35 and 36 yielded higher SLR's compared to House 33.

The plot for all 3 houses and loads appears in Fig 2.41, using method 1 - the Gordon and Zarmi Approach. The lower regions for SHF or SLR can be seen to be those for House 33, while houses 35 and 36 characterise the higher regions of the plot. A key result is the fact that the actual monitored data (hence SLR) fits the theoretical results using Gordon and Zarmi's approach (parabolic p-SLR); to the Linford House data.

These theoretical results for a direct gain Solar Heated reference house have been presented in Fig 2.11. It can be seen that the patterns are similar, though the actual observed SLR range vary, as these are climatic dependent.

Hence one could say that the approach of Gordon and Zarmi could be effectively used in monitoring the performance of actual systems.

As information was available for the auxilliary number of Kwh's employed in heating room air for each of the 3 houses (Period Oct.82-April 83) heating season; Gordon and Zarmi's approach has been used in determining the auxilliary energy consumption of each house per month from SLR and SHF weather data. The auxilliary energy consumption (QAUXCGZ) for space heating as computed by Gordon and Zarmi's method in Kwh's is shown plotted against the actual measured auxilliary energy consumption (QAUXM) for each of Houses 33,35 and 36 in Figs 2.35 2.37 and 2.39 respectively. The three plots show that House 33 with lower SHF's auxilliary heating energy consumption per month is generally higher than Houses 35 and 36 with higher SHF's, as should be expected.

The near symmetry of the three plots is an indication of the fact that the analytical techniques of Gordon and Zarmi predict the auxilliary space heating energy with a reasonable level of accuracy. (See the plot for all 3 houses, Fig 2.40), The model appears to over-estimate auxilliary energy requirements. This region in plot (2.40) is relatively small. For almost all of the data sets the model accurately predicts the auxilliary energy requirements. Also it must be noted that the load ratios employed in these computations $(\bar{L}/\bar{L}d)$ were hypothetical values as opposed to the actual values. It could be pointed out that it seems safe that the model rather than under-estimate auxilliary energy requirements in the heating season (Oct.-April), does marginally over-estimate it at periods when least auxilliary energy is needed, and again marginally under-estimates it during periods of highest needs. The later is certainly a drawback. Cowing and Kreider (20) suggested that the deviation of the estimates could result from the fact that the model assumes the insolation impinging upon a vertical south facing surface varies sinusoidally during a clear day, with a similar ambient temperature profile. The effect they suggest is to match instantaneous solar heating energy with space heating loads in a less favourable manner, than is assumed in the model. The authors in (1) and (14) also demonstrated analytically that in houses with relatively low relaxation time (~ five hours) auxilliary energy requirements are under-estimated by 6.4% Cowing and Kreider (20) further suggest a possibility of random nature error in the input measurements. All sources of input measurement errors must be eliminated or at least quantified. They sum up by suggesting estimates of instananeous and continuous measurements will permit an estimation of the uncertainties in average daily auxilliary space heating requirements for all months and or locations considered.

The main difference in the approach of Cowing and Kreider (20) from that originally pioneered by Gordon and Zarmi (1) is in the fact that the later employs a parabolic distribution function for p(SLR), the frequency of occurrence of computed SLR values. The former, however (20), employs the actual frequency distribution from typical meterological years data; a 5th order polynomial in the case of Milton Keynes. It should be expected therefore that as recommended by Gordon and Zarmi (1), Cowing and Kreider's approach yield more agreeable results. This appears to be the case comparing the computed and measured auxilliary energy requirements by both methods for all 3 houses i.e. Figs 2.40 and 2.48, about the line of symmetry. The data sets lie more evenly around the line of symmetry in the model using actual p(SLR) distributions Fig 2.48. If we now consider specific auxilliary energies of 770 Kwh/month for example, it can be seen that the two models agree exactly for House 35, fig 2.48, as opposed to Fig 2.40. The difference is however marginal.

The (real) SHF vs SLR distributions and measured vs computed auxilliary space heating requirements per month for each of House 33,35 and 36 using the 5th degree polynomial from weather data, for Linford Houses, for p(SLR) is shown in Figs 2.42 to 2.47.

From Figs 2.43, 2.45 and 2.47 deductions can be made on the effect of the load ratio ($\bar{L}/\bar{L}_d$) on the computed auxilliary energy, for each individual house; the absissa axis. It was assumed in plotting the curves that the measured auxilliary energy at each of these load ratios is non-variant; (the ordinate axis) as monitored data for various load ratios was unavailable. This also certainly affects the symmetry of the curves. If one considers the computed auxilliary energy requirement (absissa) for House 33, Fig (2.43), load ratios of ($\bar{L}_d/\bar{L}$) = 0.3 and 0.25 yield almost similar auxilliary energies. However, higher day load ratios ($\bar{L}_d/\bar{L}$) = 0.5 yield higher auxilliary energies.

Physically, this means months with higher average daily loads require higher auxilliary heating, as should be expected, for the solar inputs should be low for days with high loads. Figs 2.45 and 2.47 on the other hand for Houses 35 and 36 shows that the auxilliary energy (absissa axis) decreases with increasing daytime load ratio i.e. ($\bar{L}_d/\bar{L}$) increasing. This may be explained by the fact that days with low daytime loads, represent days with high solar inputs, but less auxilliary needs in storage from daytime for use at night time, requiring more auxilliary heating, at night time.

On the other hand, for days with high daytime loads, the auxilliary inputs during daytime for use at night time when much heating is required is higher although solar input during daytime is low, resulting in less auxilliary needs at night. The effectiveness of storage and solar inputs into the thermal mass therefore plays a role in auxilliary energy requirements.

A balance is made here between the solar inputs to storage and the auxilliary heat input to storage used up at night time, which is itself an indication of building performance. Residual energy from daytime used during night time comes into play.

It appears that the auxilliary energy needs for each of the three test cell houses favour House 35 over House 36, in turn over House 33. See the range of the absissa axis of figs 2.45, 2.47 and 2.43.

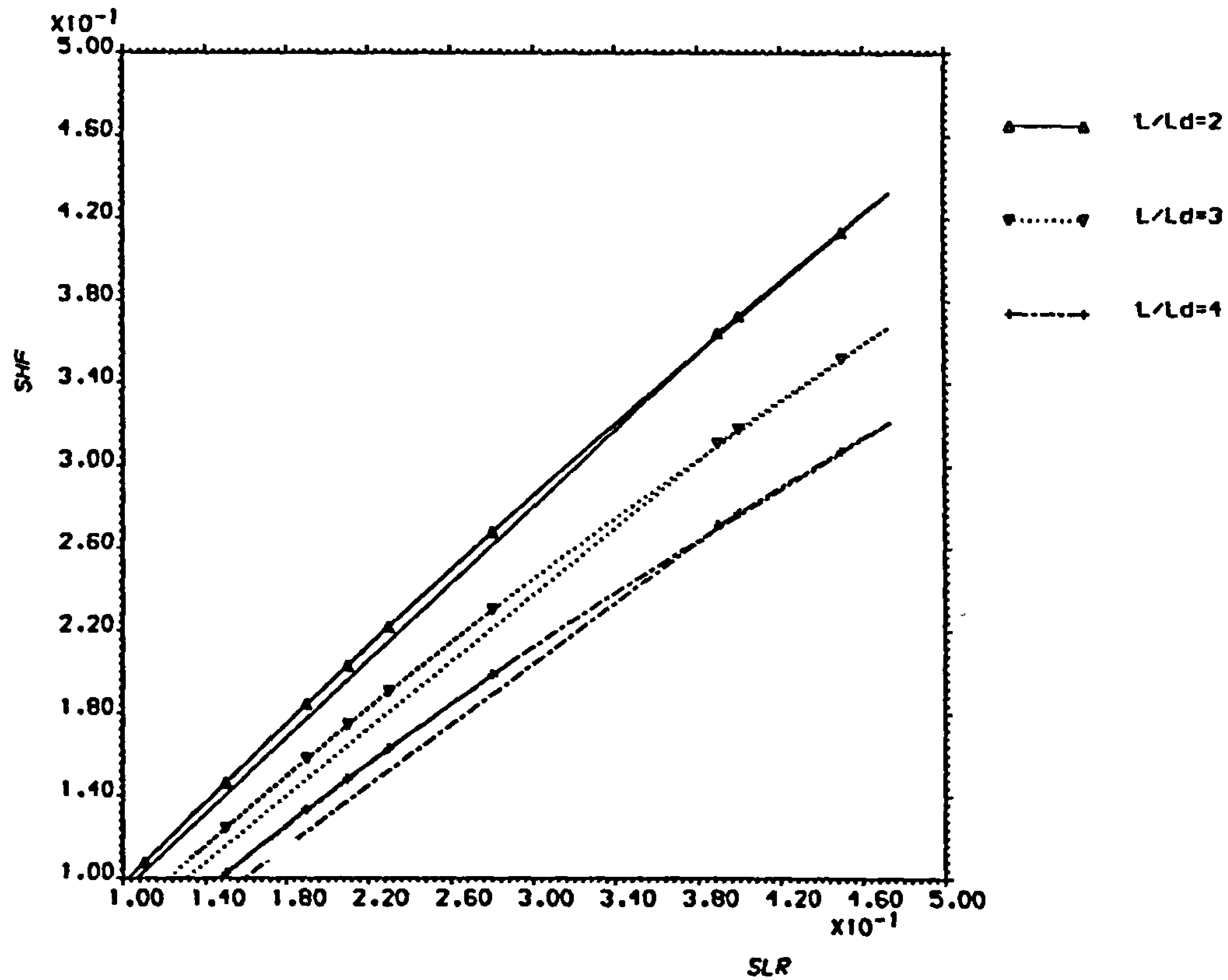


Fig 2.34 SHF vs SLR House 33 Data (Linford House ,MILTON KEYNES) . May 82 to April 83
Gordon and Zarmi Approach

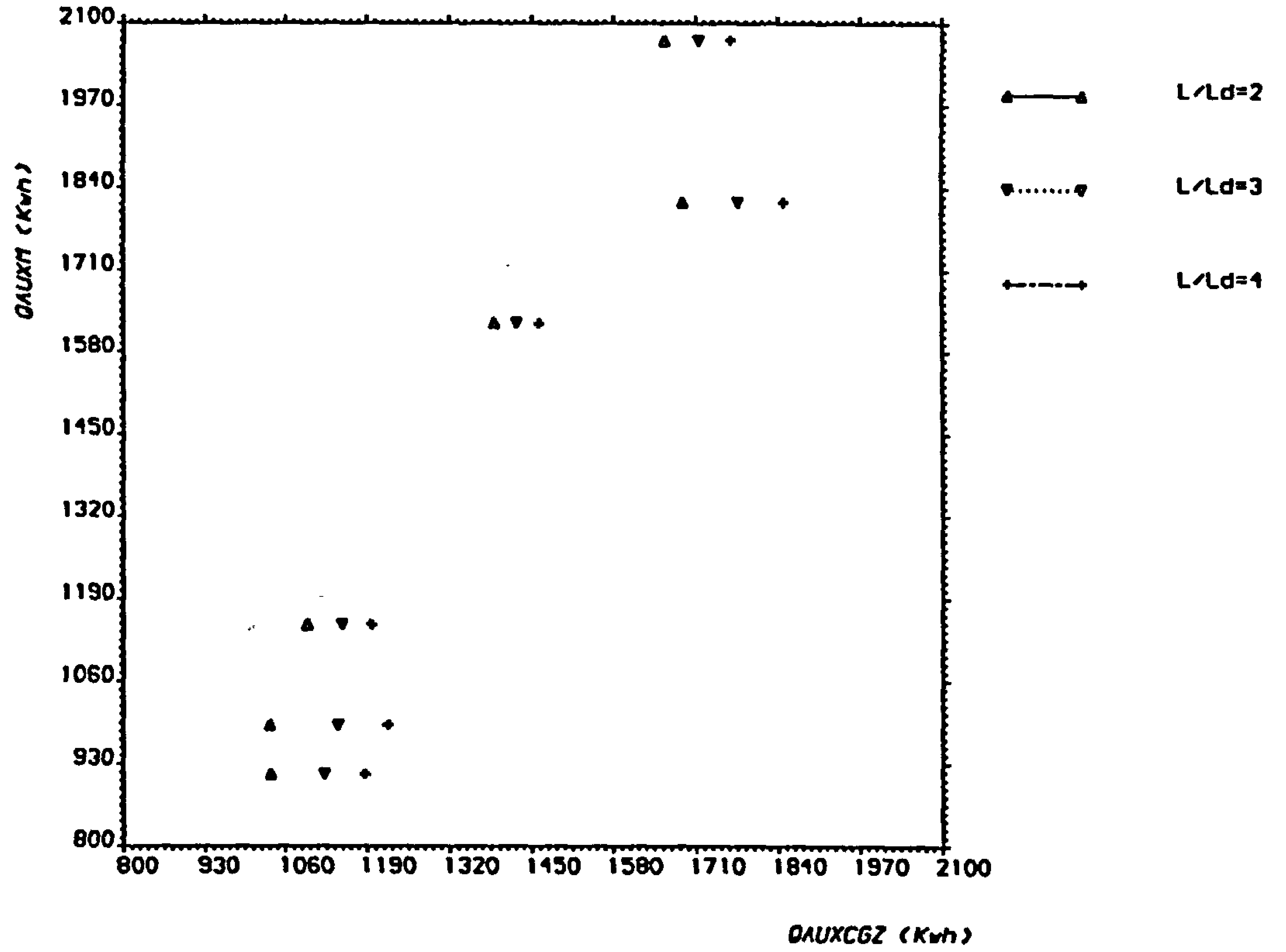


Fig 2.35 House 33 (Linford,MK) Eff. Boiler 59%:Conv.Losses 70%. QAUX Measured vs Computed (Gordon & Zarmi approach)

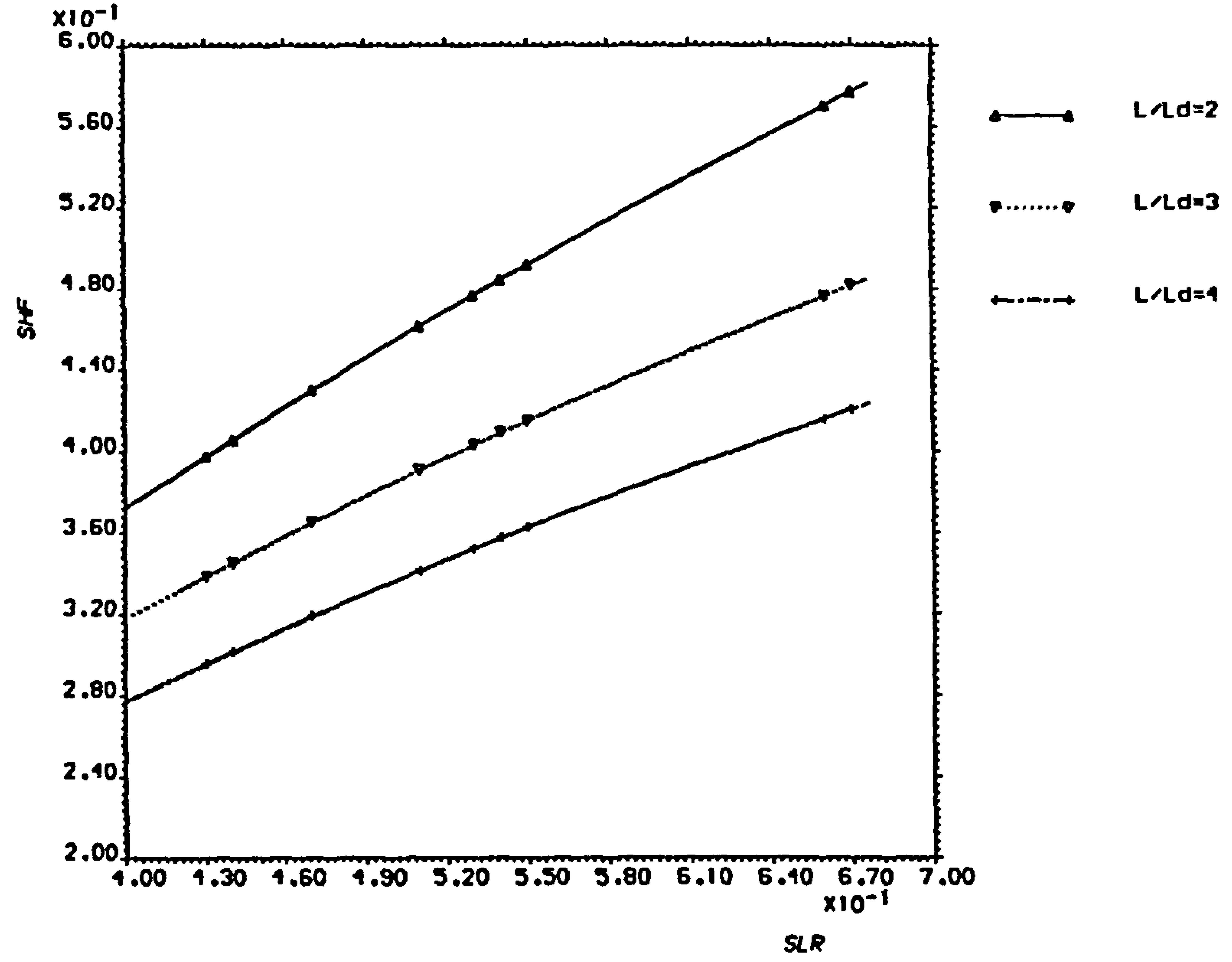


Fig.2.36 SHF vs SLR House 35 Data (Linford House, Milton Keynes) (Gordon & Zarmi) Approach.

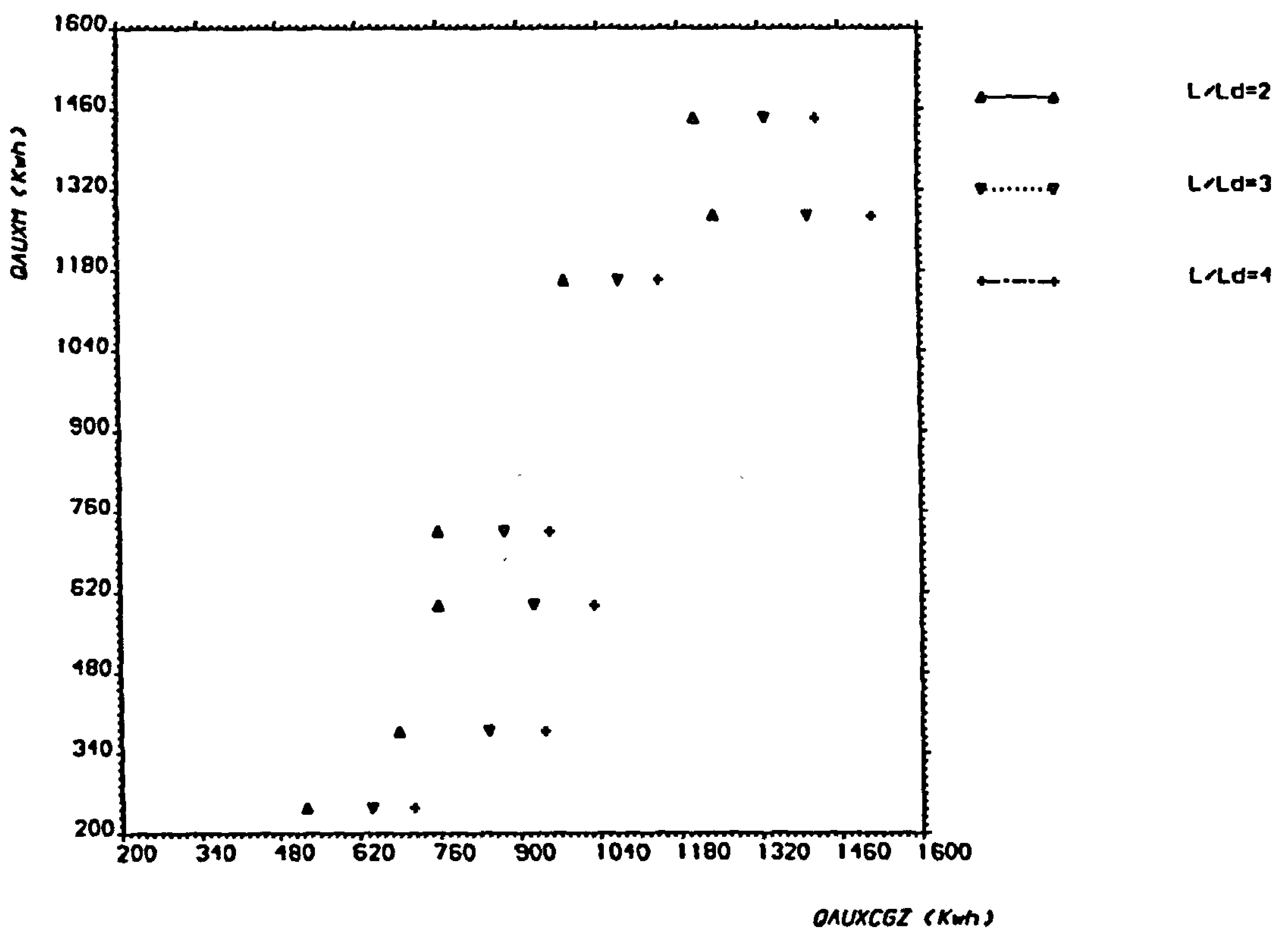


Fig. 2.37 QAUX (Measured) vs QAUXCGZ (Computed By Gordon $ Za in Kwh For Linford House 35. Compare with Fig 2.35 , House 33

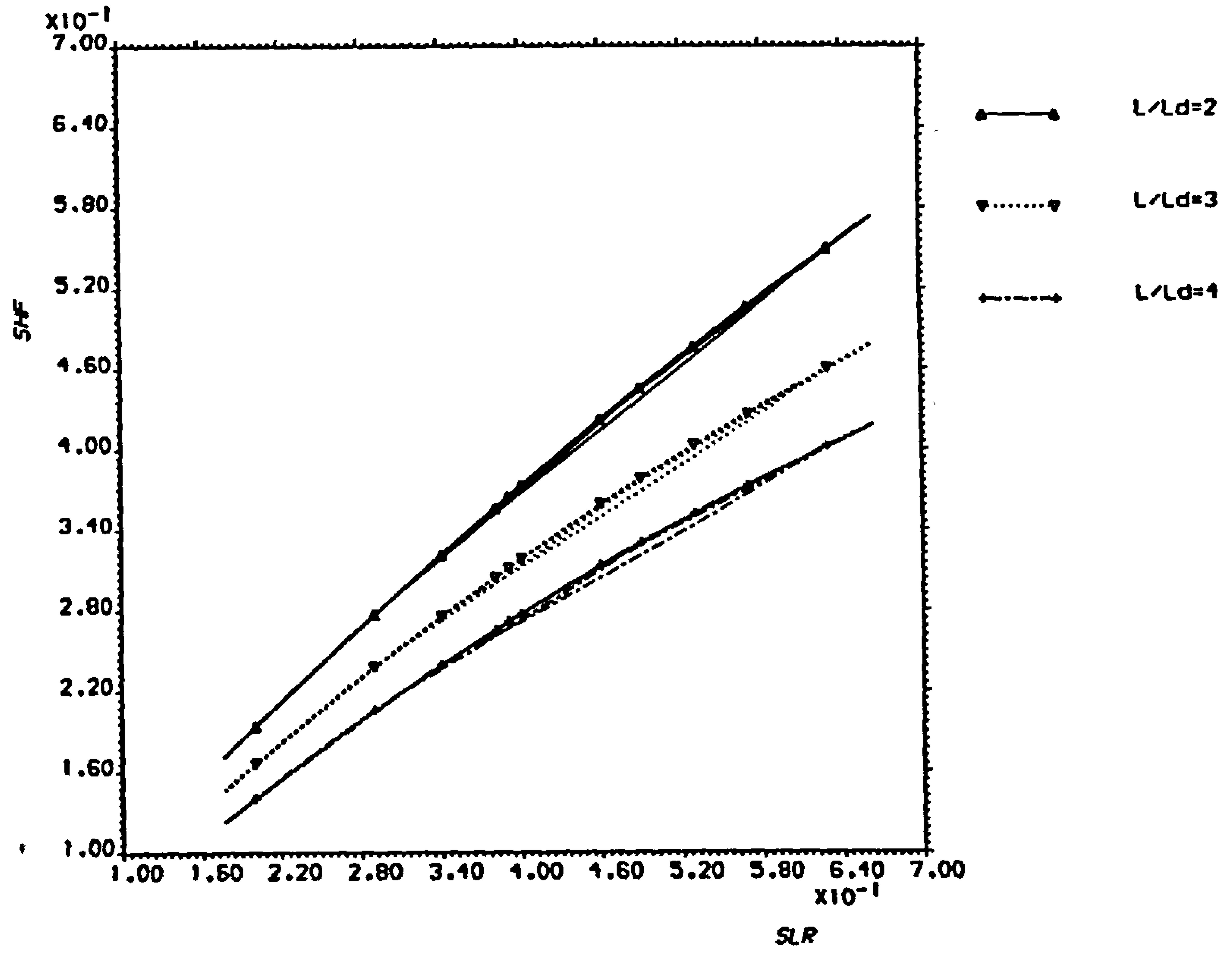


Fig. 2.38. SHF vs SLR For Linford Passive Solar House 36.
The Approach Of Gordon And Zarmi. Compare Fig 2.34 & 2.36

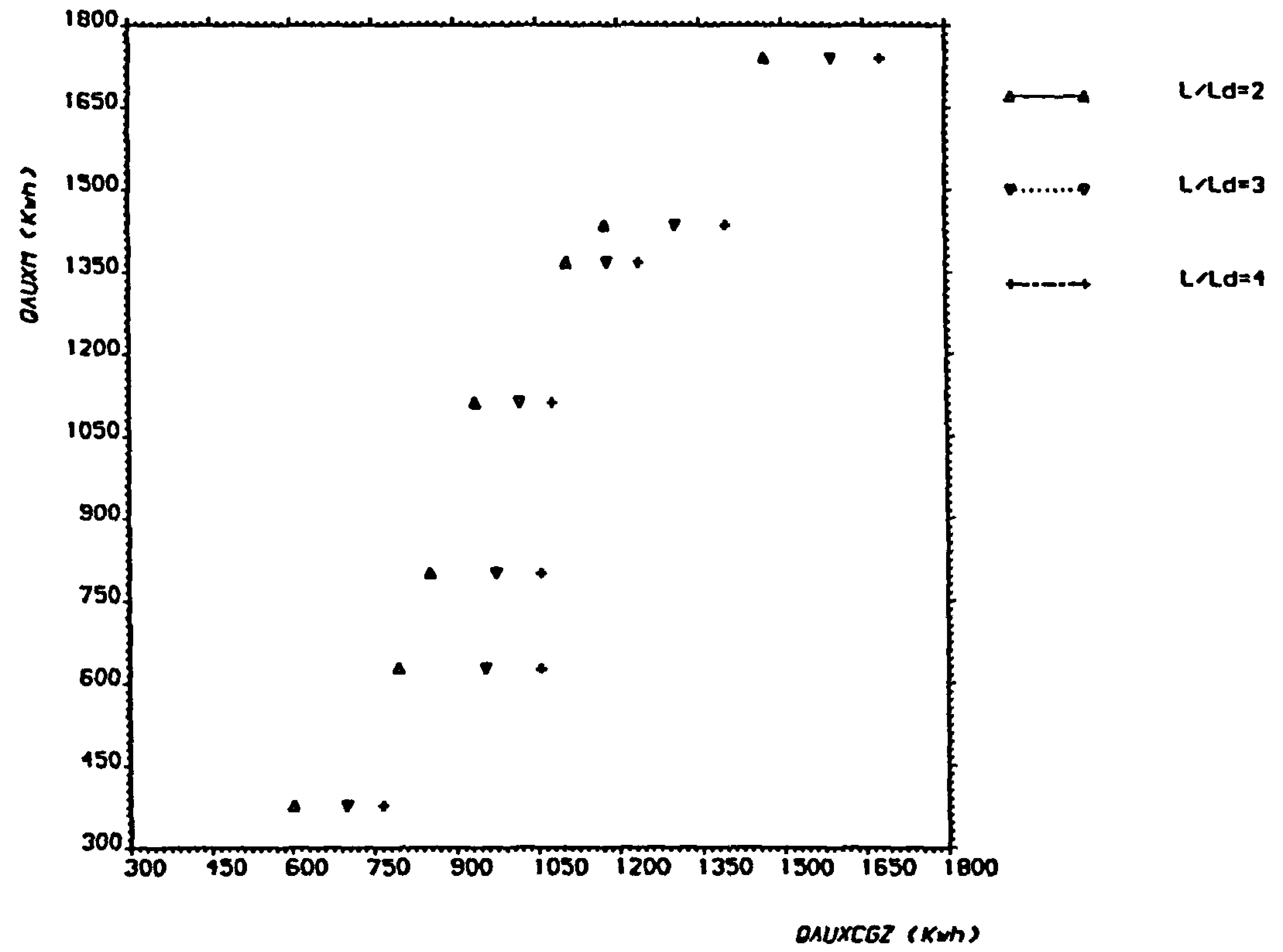


Fig. 2.39. The Measured vs Computed Auxilliary Energy Consumption In Kwh For Linford House 36. G $ Z Approach.

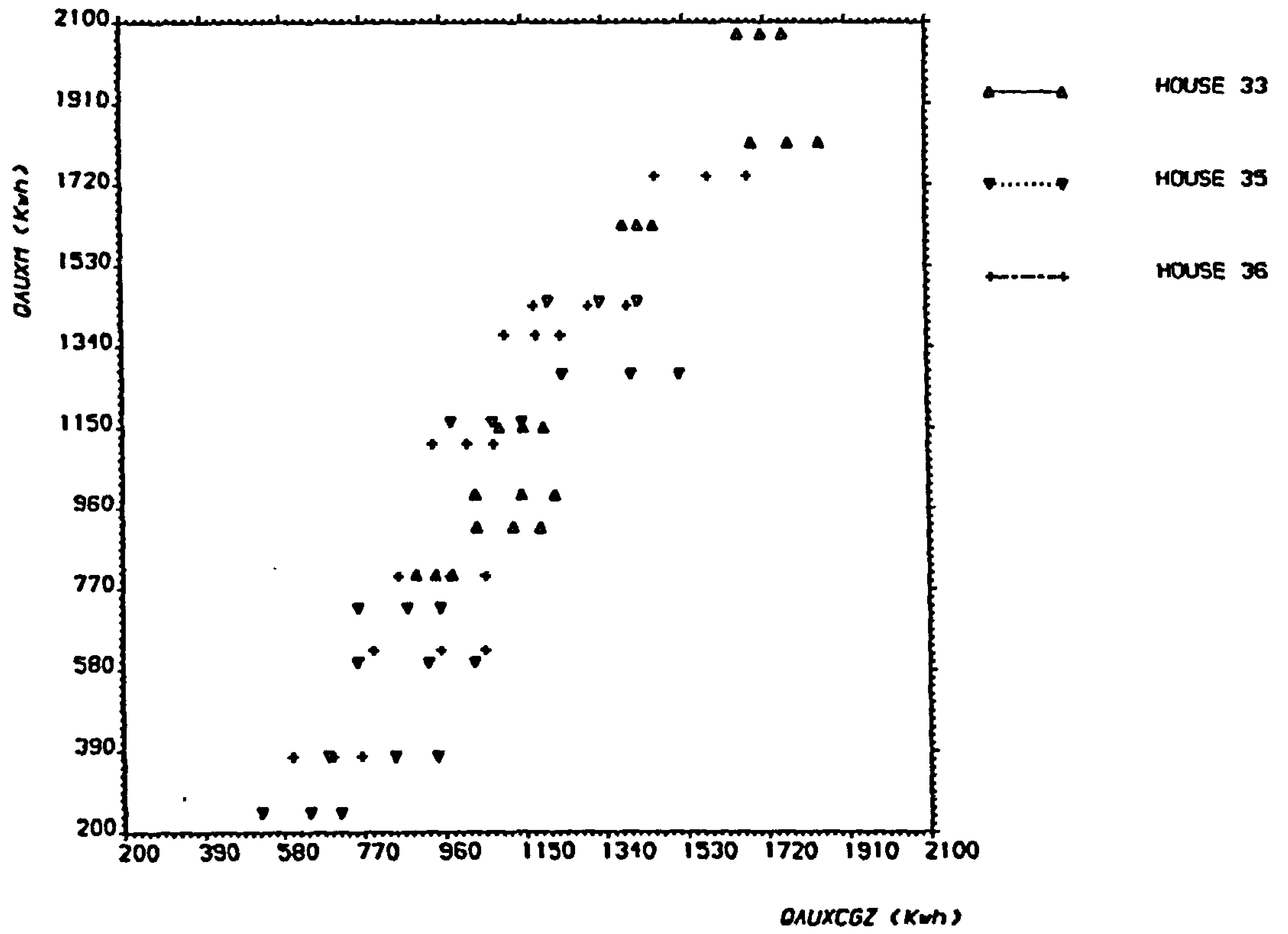


Fig. 2.40. The Measured vs Computed Auxilliary Space Heating Energy In Kwh For Linford Houses 33,35,&,36, Milton Keynes; G $ Z Appro

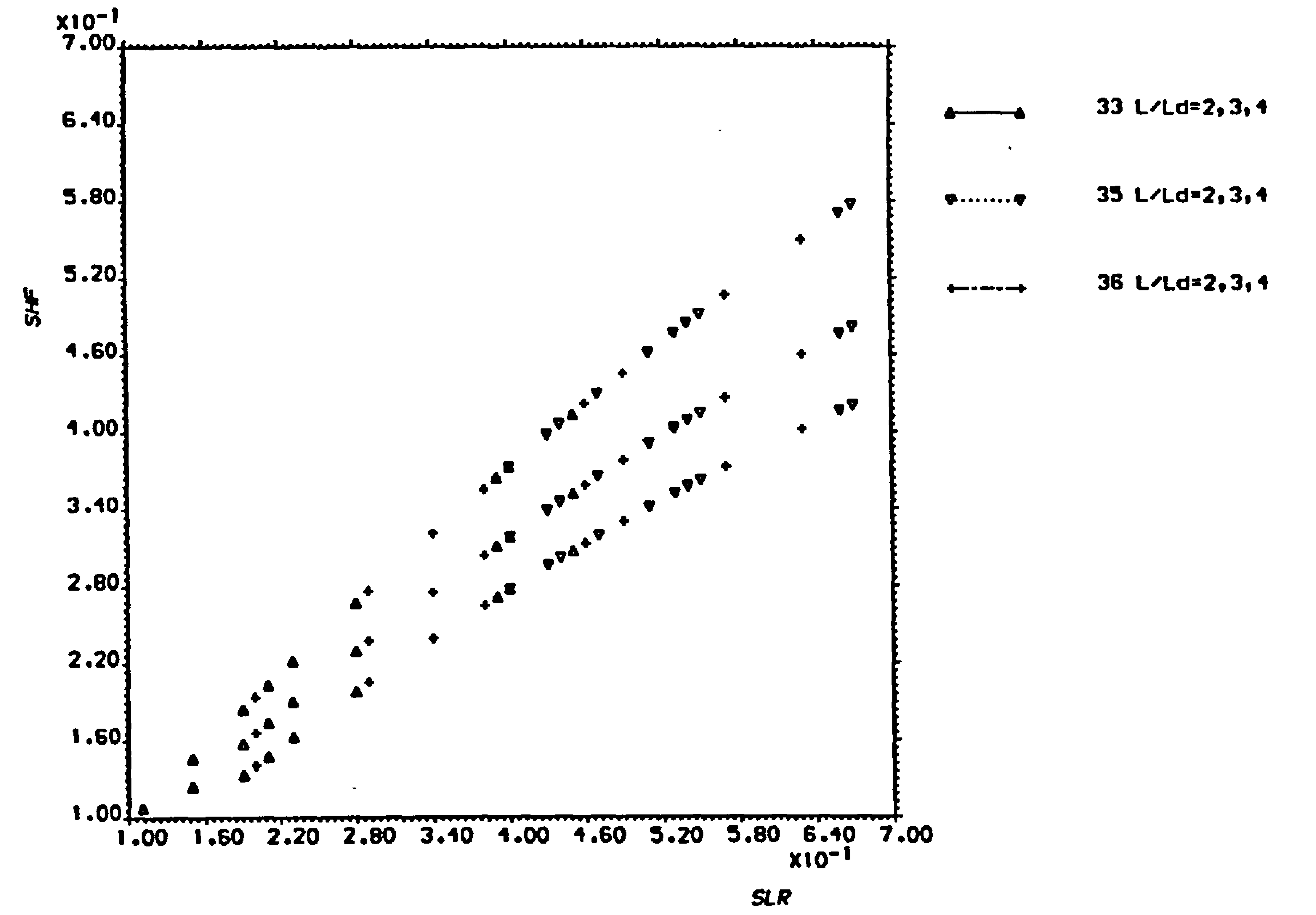


Fig. 2.41. SHF vs SLR For 3 Test Cell Linford Passive Solar Houses; The Approach Of Gordon And Zarmi. Houses 33,35 & 36.

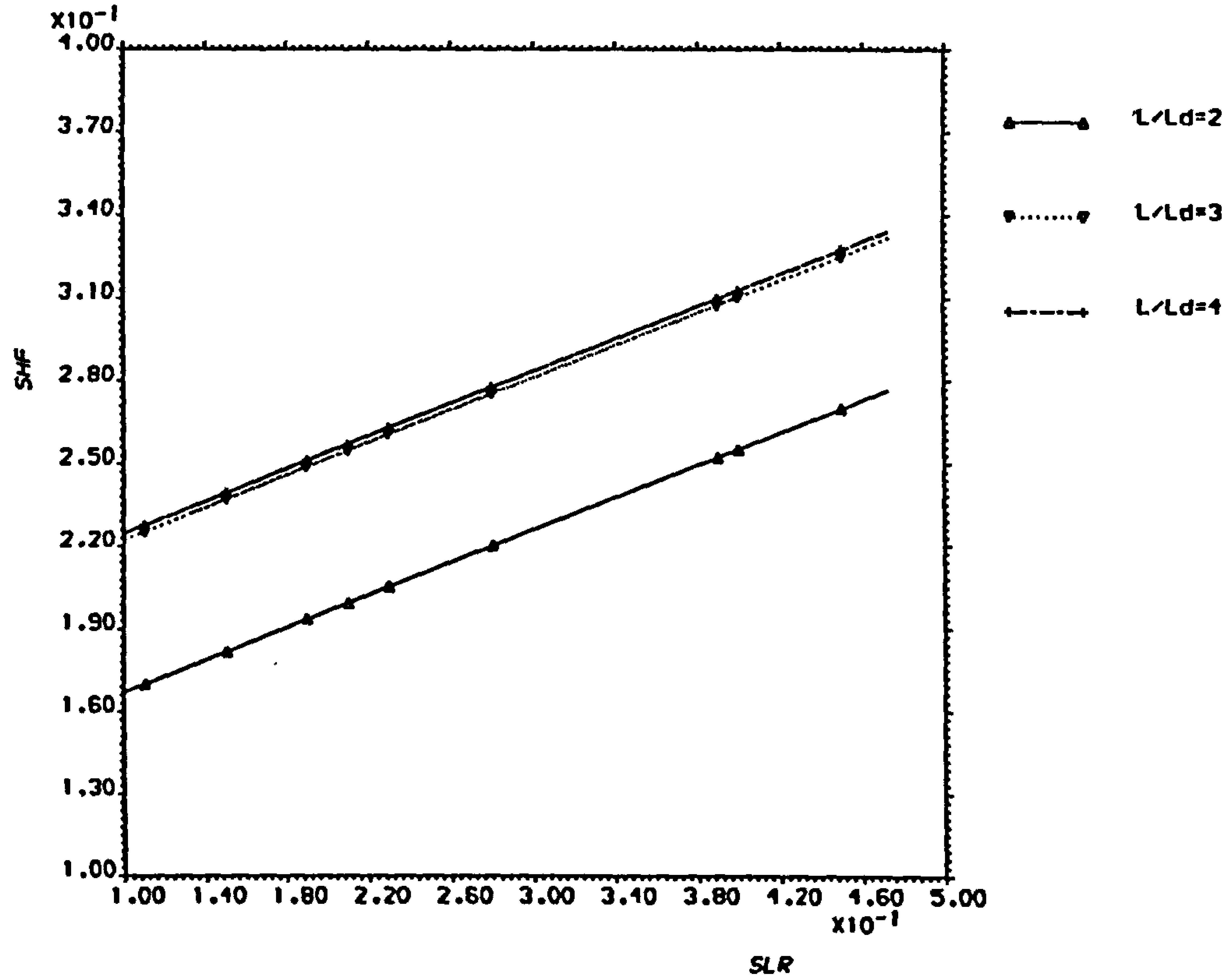


Fig. 2.42 SHF vs SLR House 33 Data (Linford House ,M.K) May 82 to Ap83
(Cowing & Krieder Approach) Using p(SLR).

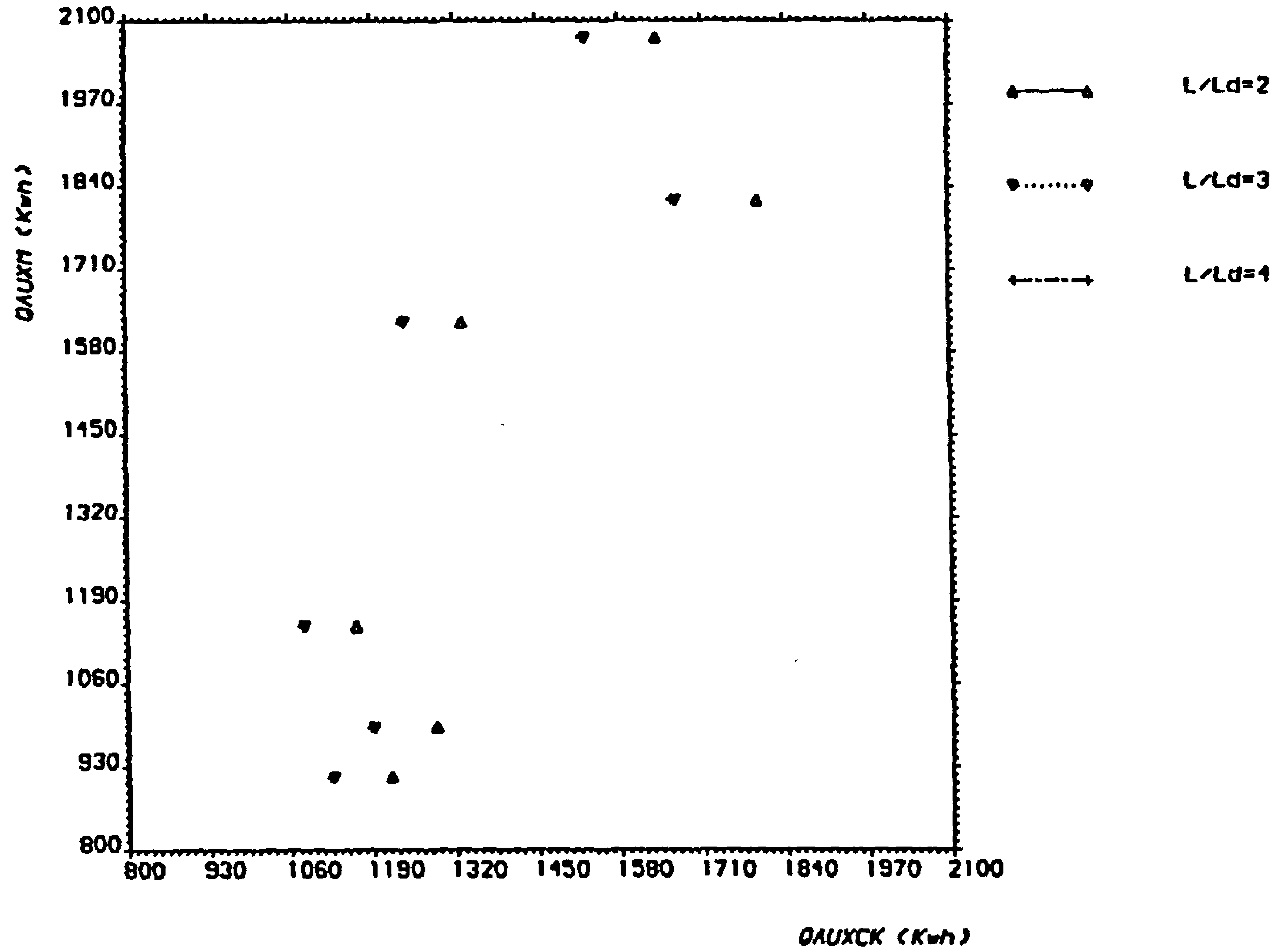


Fig 2.43 House 33 (Linford,MK) Eff. Boiler 59%:Conv.Losses 70%
QAUX Measured vs Computed (Cowing & Krieder approach)

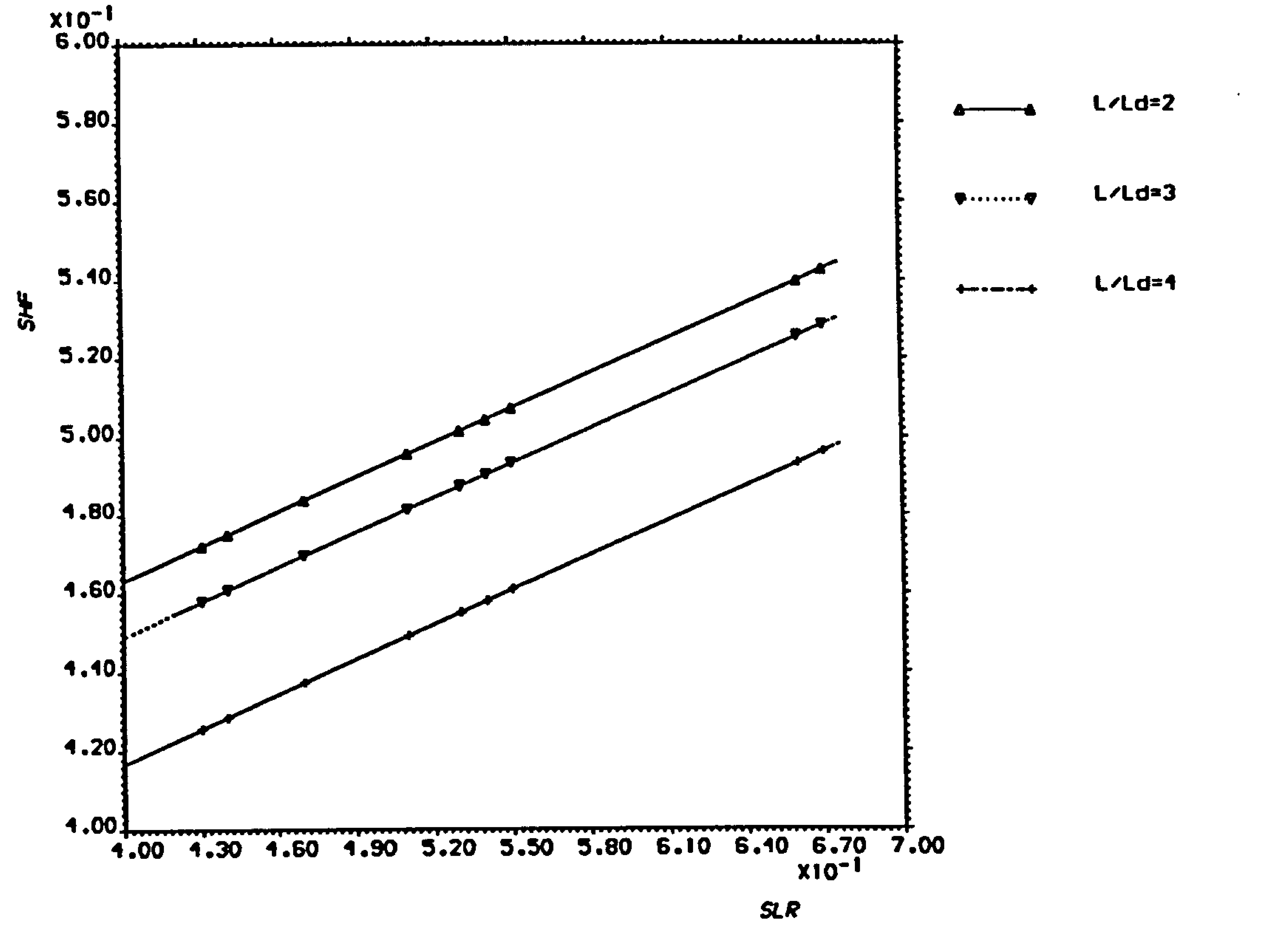


Fig.2.44 SHF vs SLR House 35 Data (Linford House, Milton Keynes) (Cowing & Krieder) Approach.

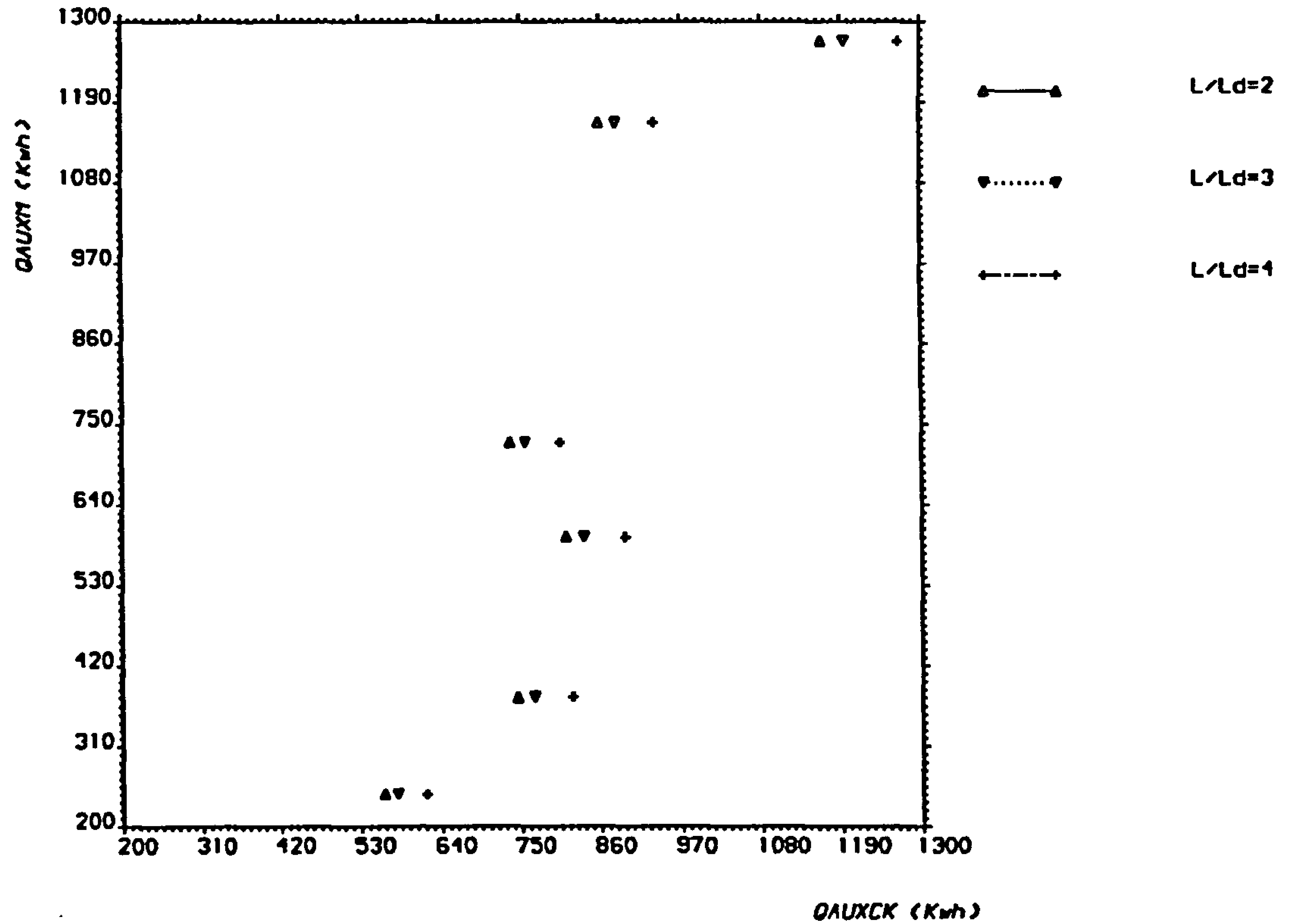


Fig. 2.45. QAUX (Mes'd) & QAUXCK (Comp. By Cowing & Krieder Approach in Kwh For Linford Hous 35. Cparé with Fig 2.43 , House 33.

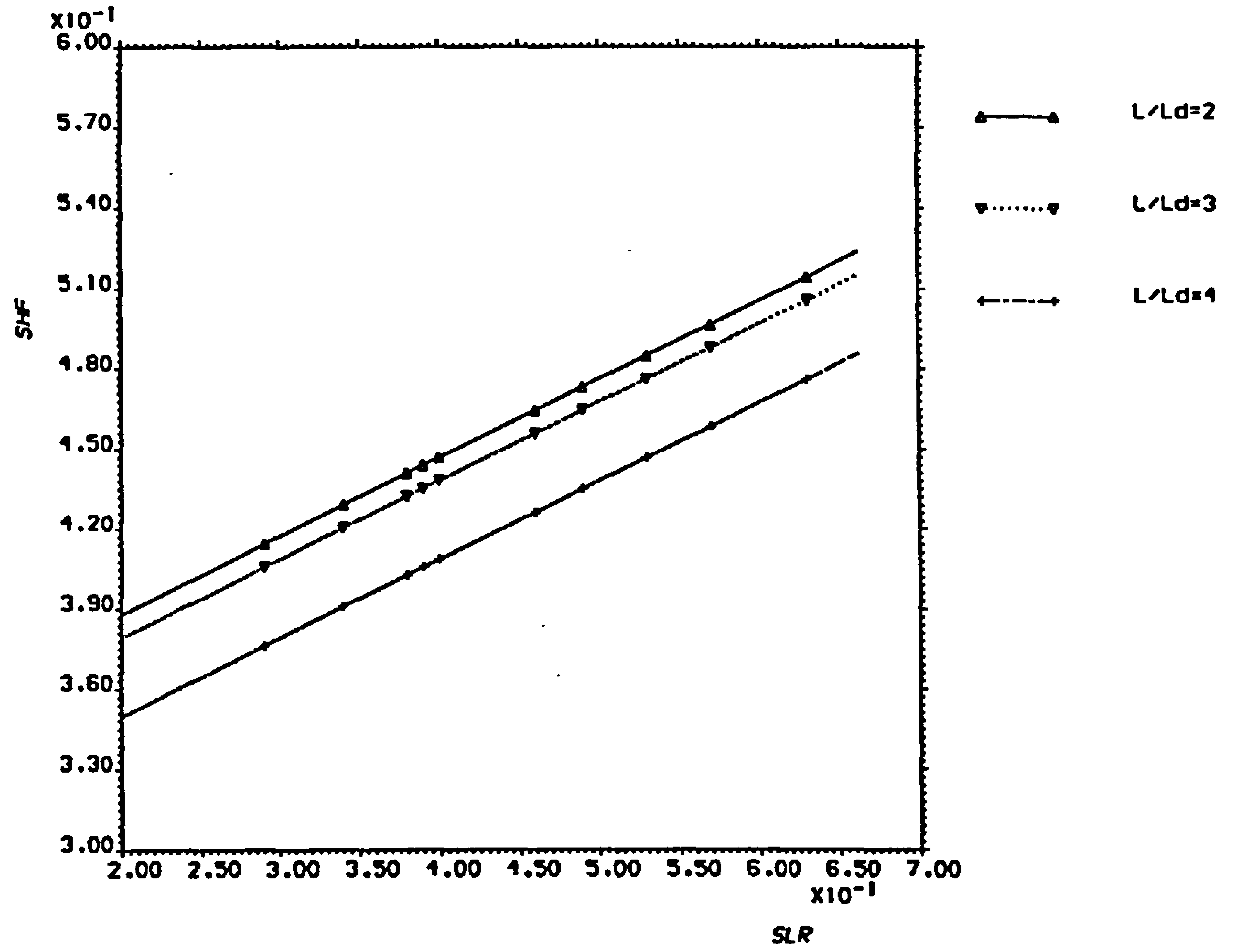


Fig. 2.46. SHF vs SLR For Linford Passive Solar House 36.
The Approach Of Cowing & Krieder. Compare Fig 2.42 & 2.44

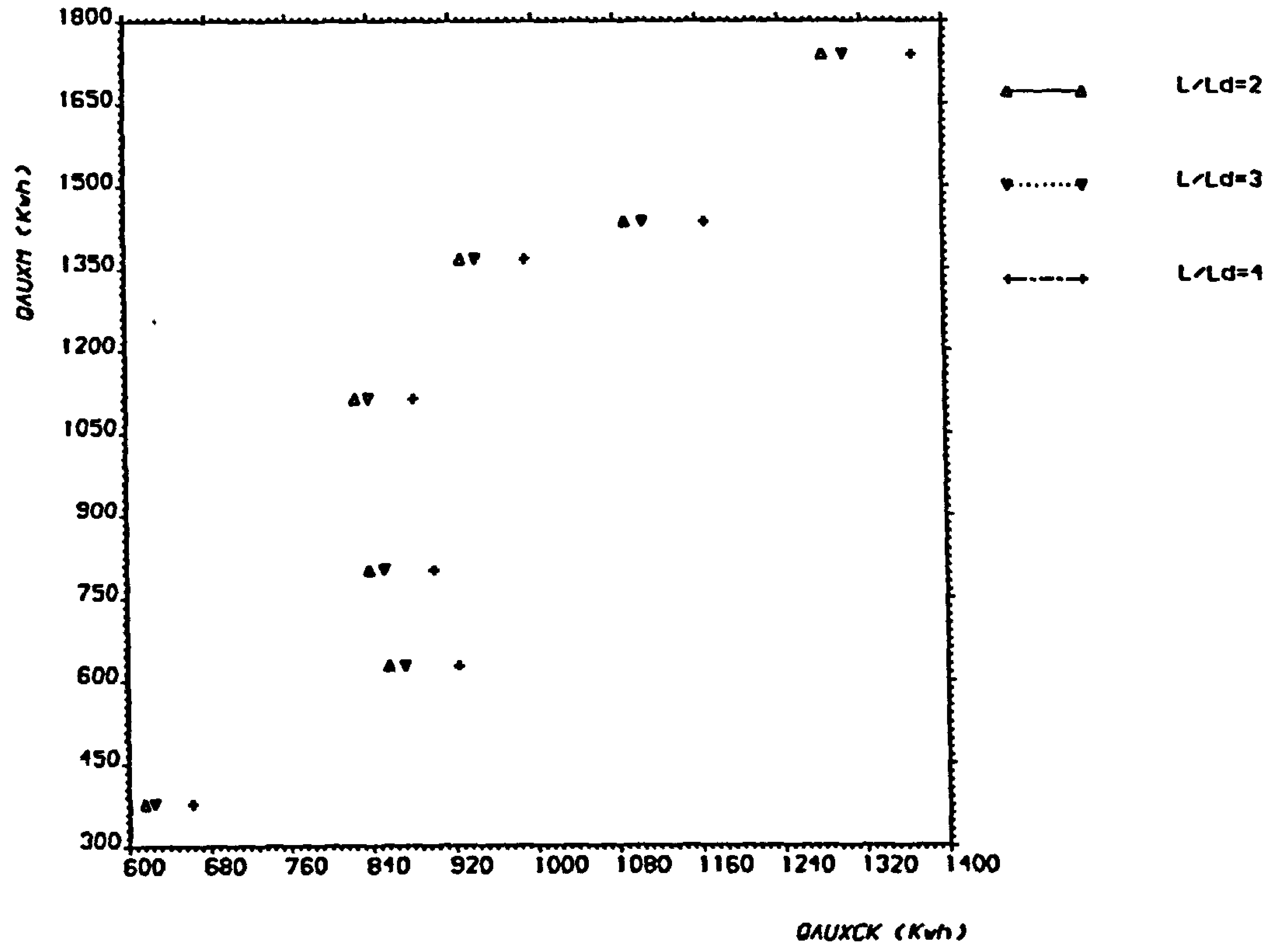


Fig. 2.47. The Measured vs Computed Auxilliary Energy Consumption In Kwh For Linford House 36. C & K Approach.

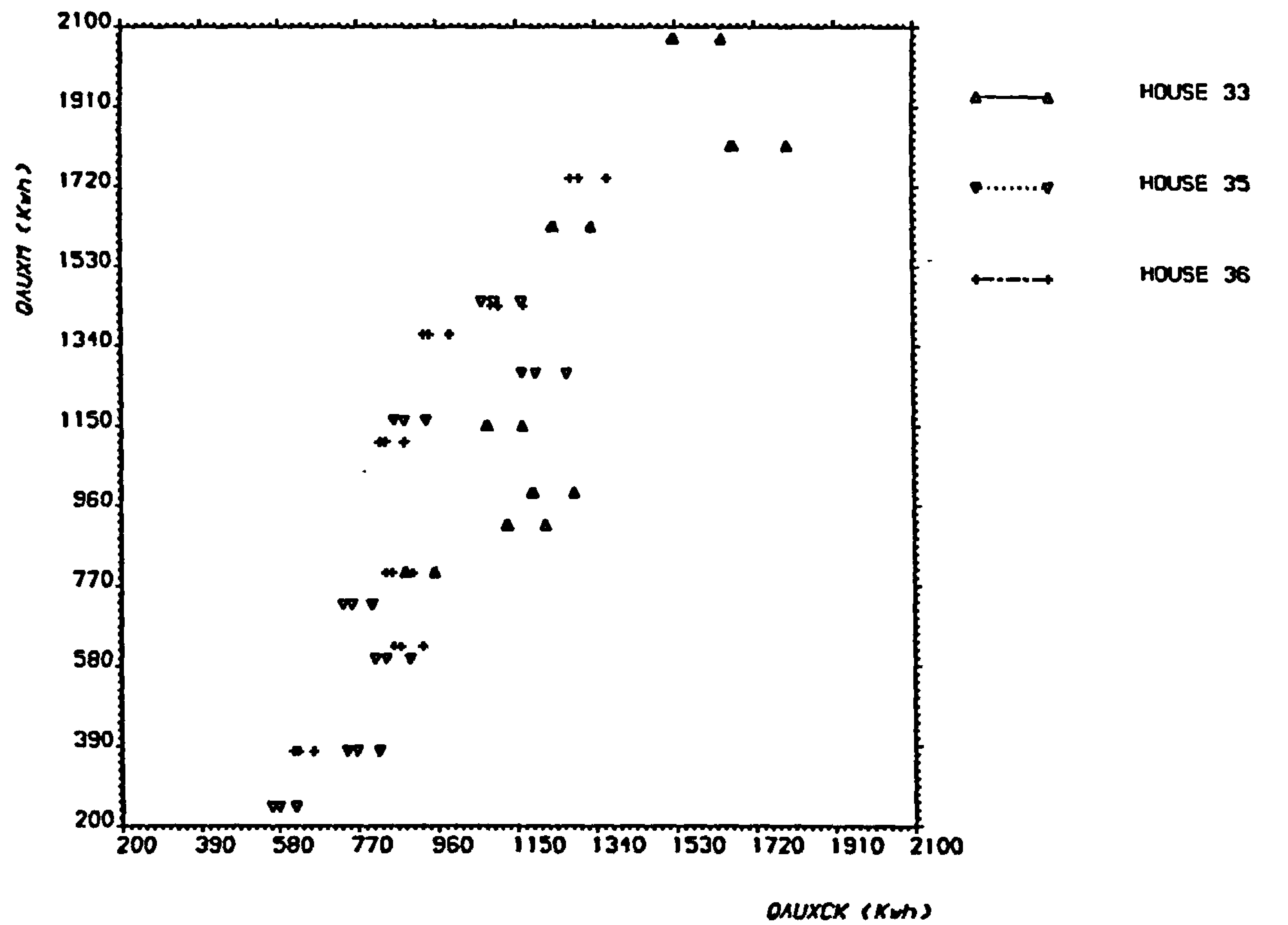


Fig. 2.48. The Measured vs Computed Auxilliary Space Heating Energy In Kwh For Linford Houses 33,35,&,36, Milton Keynes; C & K Approach

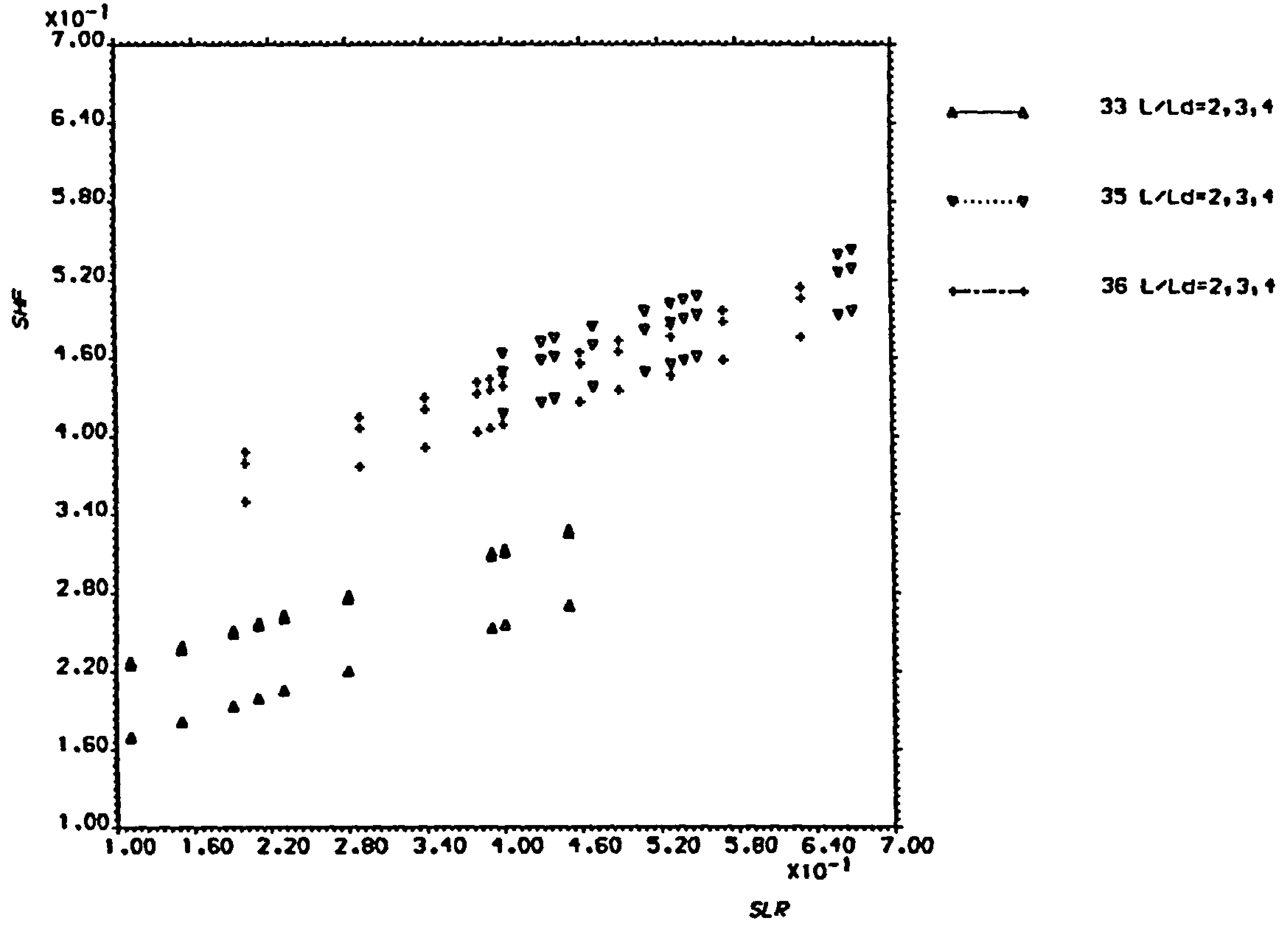


Fig. 2.49. SHF vs SLR For 3 Test Cell Linford Passive Solar Houses; The Approach Of Cowing & Krieder. Houses 33,35 & 36.

SECTION 5

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

A simple analytic method for calculating the monthly and seasonal thermal performance of passively-heated solar houses has been presented in closed form.

Both the formalism for a direct gain house with and without an attached conservatory have been presented.

Furthermore, typical reference year meteorological data for the Linford Passive solar houses, Milton Keynes and Kew Gardens, London (10 years hourly ref), are used in determining the actual SLR-distribution functions for both locations.

The methods employed for the formalism for the attached conservatory have been developed from first principles in the study. The results are compared with the simple direct gain cases also considered. Some apparent results occur.

Firstly, it should be mentioned that the main advantage of the analytic technique is that it provides a useful tool, for easy calculation, of the dependence, of thermal performance, on buildings and climatic factors. Detailed study of factors as conservatory size, ventilation rate into the building, and so on, have been undertaken. The conservatory temperature T_{co}, heat removal factor, of building envelop, F_R , and ventilation and infiltration flow rates into the building, have been studied.

The conclusions from the discussions in Section 4 show fundamentally that the introduction of the conservatory to the same ref. building (the simple direct gain house), introduces higher SHF values.

From Figs. 2.28, it was concluded that for conservatory temperatures above ambient, the conservatory temperature increases with the conservatory size B with respect to the building.

From Figs 2.28 it was depicted that the difference in temperature between a simple direct gain feature B = 0, and green-house B = 1.0, at same weather conditions is about 6.5°C.

It was demonstrated from this study, see Figs 2.24, that for low ventilation or infiltration rates, the temperature difference for various conservatory sizes is much more marked than at higher ventilation or infiltration rates. Also, for any given ventilation or infiltration rates, larger conservatories generally represent higher temperatures.

The heat removal factor F_R decreases in increasing conservatory temperature T_{co} . At $T_{co} = T_a$, F_R becomes unity. See Fig. 2.25.

Figs 2.31 illustrates the fact that the conservatory temperature decreases with increasing infiltration rates into the conservatory.

Using typical years reference data for 1) the Linford passsive-solar houses, Milton Keynes and 2) Kew Gardens, London (10 yr hourly base ref.) solar to load ratio - SLR - values; were computed for both locations, and the frequency of occurrence of each SLR value determined from the meteorological and building data, i.e ρ(SLR). The ρ - distributions for both locations is shown in Figs. 2.32 and 2.33.

A 5th Degree polynomial was then fitted on both curves and the coefficients representing the ρ - distributions determined. A correlation order of 0.97 and 0.98 represented a good fit, as in the work by Kreider and Cowing (20), thus validating the approach to two U.K. locations.

A further extension of this work will involve the compilation of data for about 40 houses and estimating the daily auxilliary energy from this approach; then, comparing with measured data as in ref. (20). The case of an attached solarium can also be considered.

As an indication of the practicability and validation of the analytical techniques detaily described in this paper, the case of 3 real direct gain houses was considered. The houses are of the direct gain type and are described as Houses 33,35 and 36. They are the Linford Houses in the Milton Keynes District in England. Meteorological data and records of the auxilliary space heating energy usage over the heating season (Oct.82-April 83, 7 months) has been compared to computations by each of two methods.

The analysis is carried out for these three houses independently and collectively using the two methods. Method (A) assumes a parabolic distribution function for SLR, p(SLR), given by eqn. (1.3.1), sufficiently describes the frequency of occurrence of computed SLR values, over the heating seasons 7 months' duration. Method (B) uses the real statistically determined frequency distribution p(SLR) from the weather data. A 5th order polynomial sufficiently described all p(SLR) values with good correlation ($r = 0.97$), Fig 2.32.

The average predictions of auxilliary energy usage in (Kwh/month) by model (B) using actual p-SLR weather data appears to favour observed records more than Model (A) using a parabolic p-SLR, as recommended by Gordon and Zarmi[1].

As many houses in the U.K. use conservatories to further enhance building performance the rigorous equations derived in this report for such situations will obviously be of immense use in designing buildings in the U.K and Europe at large. Finally, it is hoped that the analytical results presented will help understand the physics of the study.

CHAPTER 6

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REFERENCES

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CHAPTER 7

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PUBLISHED REPORTS

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CHAPTER 8

APPENDICES

A1. Solution of Heat Balance Equation for Direct Gain House, EQN. 1.3.9

From eqn. (1.3.9)

$$(mC)_s \, dT_s^d / dt^d = \frac{(1-F)}{\Delta t^d} Q_T - U_{sA} (T_s^d - T_c) \qquad (A\ 1.1)$$

We denote the storage mass $(mC)_s = D$,
$\frac{(1-F)Q_T}{\Delta t^d} = A$, $U_{sA} = B$, $T_s^d = y$, $t^d = x$, and $T_c = C$
Eqn. (1.1) becomes

$$D \frac{dy}{dx} = A - B(y - C) \qquad (A\ 1.2)$$

or $$D \frac{dy}{dx} + By = A + BC \qquad (A\ 1.3)$$

writing $A + BC = K$,

$$D \frac{dy}{dx} + By - K = 0 \qquad (A\ 1.4)$$

$$\frac{dy}{dx} + \frac{B}{D} y = K/D$$

$$(\times e^{(B/D)x}) \qquad e^{\frac{B}{D}x} \frac{dy}{dx} + y \cdot \frac{B}{D} \cdot e^{\frac{B}{D}x} = \frac{K}{D} e^{\frac{B}{D}x} \qquad (A\ 1.5)$$

$$\frac{d}{dx} (y \cdot e^{\frac{B}{D}x}) = \frac{K}{D} e^{\frac{B}{D}x} \qquad (A\ 1.6)$$

integrating both sides w.r.t x

$$y \cdot e^{\frac{B}{D}x} = \frac{K \cdot e^{\frac{B}{D}x}}{D \cdot \frac{B}{D}} + c \qquad (A\ 1.7)$$

$$(\div e^{\frac{B}{D}x}) \qquad y = \frac{K}{D \cdot \frac{B}{D}} + c\, e^{-\frac{B}{D}x} \qquad (A\ 1.8)$$

$$\therefore T_s^d = \left[\frac{(1-F)Q_T}{\Delta t^d U_{sA}} + T_c \right] + c\, e^{-[U_{sA}/(mC)_s] t^d}$$

(A 1.8)

To evaluate C, $T_s^d = T_s(0)$ at $t^d = 0$

$$\Longrightarrow \quad T_s(0) - \left[\frac{(1-F)\,Q_T}{\Delta t^d\,U_{SA}} + T_c\right] = C \qquad (A1{\cdot}9)$$

$\xrightarrow{A1\cdot9 \text{ in } A1\cdot8}$

and writing $\tau = (mC)_s / U_{SA}$

$$T_s^d = \left[\cancel{\frac{(1-F)\,Q_T}{\Delta t^d\,U_{SA}}} + T_c\right] + \left\{T_s(0) - \left[\cancel{\frac{(1-F)\,Q_T}{\Delta t^d\,U_{SA}}} + T_c\right]\right\} e^{-t^d/\tau}$$

$$T_s^d - T_c = \left[\frac{(1-F)\,Q_T}{\Delta t^d\,U_{SA}}\,(1 - e^{-t^d/\tau})\right] + (T_s(0) - T_c)\,e^{-t^d/\tau}$$

OR

$$T_s^d(t^d) - T_c = \left[\frac{(1-F)\,Q_T}{\Delta t^d\,U_{SA}}\,(1 - e^{-t^d/\tau})\right] + (T_s(0) - T_c)\,e^{-t^d/\tau} \qquad (A1{\cdot}10)$$

A2 Solution of Heat Balance Equation for the Water Wall House (Eqn.1.5.1)

From eqn. 1.5.1

$$(mC)_s\, dT_s^d / dt^d = (\alpha Q_T/\Delta t^d) - U_{sa}^d (T_s^d - T_a^d) - U_{sA}(T_s^d - T_c) \qquad (A2.1)$$

We rewrite eqn. (A.2.1) as

$$D\frac{dy}{dx} = A - B(y-C) - E(y-F)$$

$$D\frac{dy}{dx} = A - y(B+E) + (BC+EF)$$

$$D\frac{dy}{dx} = I - Gy \qquad (A2.2)$$

where

$$I = A + BC + EF$$
$$G = B + E$$

From eqn. (A1.8) the solution of (A2.2) is

$$y = \frac{I}{G} + c\, e^{-\frac{G}{D}x} \qquad (A2.3)$$

$$\frac{I}{G} = \frac{\alpha Q_T}{\Delta t^d (U_{sa}^d + U_{sA})} + \frac{T_a^d U_{sa}^d}{U_{sa}^d + U_{sA}} + \frac{U_{sA} T_c}{U_{sa}^d + U_{sA}} \qquad (A2.4)$$

$$\frac{G}{D} = \frac{U_{sa}^d + U_{sA}}{(mC)_s} \qquad (A2.5)$$

(2.3) - (2.5) $\Longrightarrow$

$$T_s^d = \frac{\alpha Q_T}{\Delta t^d (U_{sa}^d + U_{sA})} + \frac{U_{sa}^d T_a^d}{U_{sa}^d + U_{sA}} + \frac{U_{sA} T_c}{U_{sa}^d + U_{sA}} + c\, e^{-[(U_{sa}^d + U_{sA})/(mC)_s]t^d} \qquad (A2.6)$$

if $\tau^d = (mC)_s/(U_{sa}^d + U_{sA})$, then substituting $T_s^d = T_s(0)$, at $t^d = 0$ in eqn (A2.6), gives,

$$T_s(0) - \frac{\alpha Q_T}{\Delta t^d (U_{sa}^d + U_{sA})} - \frac{U_{sa}^d T_a^d}{U_{sa}^d + U_{sA}} - \frac{U_{sA} T_c}{U_{sa}^d + U_{sA}} = c \qquad (A2.7)$$

A2.7 in A2.6 $\Longrightarrow$

$$T_s^d(t^d) = \frac{\alpha Q_T}{\Delta t^d (U_{sa}^d + U_{sA})} + \frac{U_{sa}^d T_a^d}{U_{sa}^d + U_{sA}} + \frac{U_{sA} T_c}{U_{sa}^d + U_{sA}}$$

$$+ \left[T_s(0) - \frac{\alpha Q_T}{\Delta t^d (U_{sa}^d + U_{sA})} - \frac{U_{sa}^d T_a^d}{U_{sa}^d + U_{sA}} - \frac{U_{sA} T_c}{U_{sa}^d + U_{sA}} \right] e^{-t^d/\tau^d} \qquad (A2.8)$$

or

$$T_s^d(t^d) = \frac{\alpha Q_T (1 - e^{-t^d/\tau^d})}{\Delta t^d (U_{sa}^d + U_{sA})} + \frac{(1 - e^{-t^d/\tau^d})(U_{sa}^d T_a^d + U_{sA} T_c)}{U_{sa}^d + U_{sA}} + T_s(0) e^{-t^d/\tau^d} \quad (A2.8)$$

Adding and subtracting $U_{sa}^d T_c (1 - e^{-t^d/\tau^d}) / (U_{sa}^d + U_{sA})$

on R.H.S of A2.8 gives

$$T_s^d(t^d) = \frac{\alpha Q_T (1 - e^{-t^d/\tau^d})}{\Delta t^d (U_{sa}^d + U_{sA})} - \frac{U_{sa}^d (1 - e^{-t^d/\tau^d})(T_c - T_a^d)}{U_{sa}^d + U_{sA}}$$

$$+ \frac{U_{sa}^d T_c (1 - e^{-t^d/\tau^d})}{U_{sa}^d + U_{sA}} + \frac{U_{sA} T_c (1 - e^{-t^d/\tau^d})}{U_{sa}^d + U_{sA}} + T_s(0) e^{-t^d/\tau^d} \quad (A2.9)$$

The 3rd and 4th terms added gives

$(T_c - T_c e^{-t^d/\tau^d})$.

Hence eqn. (A2.9) becomes

$$T_s^d(t^d) - T_c = \frac{\alpha Q_T (1 - e^{-t^d/\tau^d})}{\Delta t^d (U_{sa}^d + U_{sA})} - \frac{U_{sa}^d (1 - e^{-t^d/\tau^d})(T_c - T_a^d)}{U_{sa}^d + U_{sA}} + (T_s(0) - T_c) e^{-t^d/\tau^d} \quad (A2.10)$$

with τ^d given by

$$\tau^d = (mC)_s / (U_{sa}^d + U_{sA}) \quad (A2.11)$$

A3. Gordon and Zarmi (14) illustrated that the non-linear heat diffusion through the massive storage wall can be treated as a linear one node model, for wall thicknesses of practical interest.

EQN.

A4. SOLUTION OF THE HEAT BALANCE IN DAYTIME

The daytime heat balance equation for a direct gain building with attached conservatory is

$$(mC)_s \, dT_s^d / dt^d = (1-B)\left[\frac{(1-F)\,Q_T}{\Delta t^d} - U_{SA}(T_s^d - T_c)\right] + B\left[U_{co,s}(T_{co}^d - T_s^d) + \frac{(1-F_c)\,Q_T}{\Delta t^d}\right]$$

writing

$$(mC)_s = D \quad , \quad T_s^d = y \quad , \quad t^d = x$$

$$A = \frac{(1-B)(1-F)\,Q_T}{\Delta t^d}$$

$$B^* = (1-B)\,U_{SA}$$

$$C = T_c$$

$$E = B \cdot U_{co,s}$$

$$T_{co}^d = F$$

$$G = \frac{(1-F_c)\,Q_T\,B}{\Delta t^d}$$

Appendix 4

the equation can be written as

$$D\frac{dy}{dx} = A - B^*(y - C) + E(F - y) + G$$

$$\therefore D\frac{dy}{dx} = (A + B^*C + EF + G) - (B^* + E)y$$

$$\therefore D\frac{dy}{dx} + B_{A1}\,y = K$$

$$\frac{dy}{dx} + \frac{B_{A1}}{D}\,y = \frac{K}{D}$$

solution from Appendix 1 is

$$y = \frac{K}{B_{A1}} + C^* e^{(-B_{A1}/D)x} \qquad (1)$$

where

$$K = (A + B^*C + EF + G)$$

$$= \frac{(1-B)(1-F)Q_T}{\Delta t^d} + (1-B)U_{SA}\cdot T_c + B\cdot U_{co,s}T_{co}^d + \frac{(1-F_c)Q_T\,B}{\Delta t^d}$$

and $B_{A1} = B^* + E = (1-B)U_{SA} + BU_{co,s}$

$$\Rightarrow K/B_{A1} = \frac{(1-B)(1-F)Q_T}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} + \frac{(1-B)U_{SA}T_c}{(1-B)U_{SA} + BU_{co,s}} + \frac{B\cdot U_{co,s}T_{co}^d}{(1-B)U_{SA} + BU_{co,s}} + \frac{(1-F_c)Q_T\,B}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} \qquad (2a)$$

and

$$\frac{B_{A1}}{D} = \frac{(1-B)U_{SA} + BU_{co,s}}{(mC)_s} \qquad (2b)$$

2a & 2b in 1 $\longrightarrow$

$$T_s^d(t^d) = \frac{(1-B)(1-F)\,Q_T}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} + \frac{(1-B)\,U_{SA}\cdot T_c}{(1-B)U_{SA} + BU_{co,s}}$$

$$+ \frac{B\,U_{co,s}\;T_{co}^d}{(1-B)U_{SA} + BU_{co,s}} \quad \frac{(1-F_c)\,Q_T\;B}{\Delta t^d((1-B)U_{SA} + BU_{co,s})}$$

$$+ C^* e^{-[((1-B)U_{SA} + BU_{co,s})/(mC)_s]t^d} . \qquad (3)$$

If $\tau^d = (MC)_s/((1-B)U_{SA} + BU_{co,s})$,
substituting $T_s^d = T_s(0)$ at $t^d = 0$;
then equation (3) becomes

$$T_s(0) - \frac{(1-B)(1-F)\,Q_T}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} - \frac{(1-B)\,U_{SA}\,T_c}{(1-B)U_{SA} + BU_{co,s}}$$

$$- \frac{B\,U_{co,s}\;T_{co}^d}{(1-B)U_{SA} + BU_{co,s}} - \frac{(1-F_c)\,Q_T\;B}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} = C^* . \qquad (4)$$

(4) in (3) $\longrightarrow$

$$T_s^d(t^d) = \Big[\frac{(1-B)(1-F)Q_T}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} + \frac{(1-B)\,U_{SA}\,T_c}{(1-B)U_{SA} + B\cdot U_{co,s}} +$$

$$\frac{B\cdot U_{co,s}\;T_{co}^d}{(1-B)U_{SA} + B\cdot U_{co,s}} + \frac{(1-F_c)\,Q_T\;B}{\Delta t^d((1-B)U_{SA} + B\cdot U_{co,s})}\Big] +$$

$$\Big[T_s(0) - \frac{(1-B)(1-F)Q_T}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} - \frac{(1-B)\,U_{SA}\;T_c}{(1-B)U_{SA} + BU_{co,s}} -$$

$$\frac{B\;U_{co,s}\;T_{co}^d}{(1-B)U_{SA} + BU_{co,s}} - \frac{(1-F_c)\,Q_T\;B}{\Delta t^d((1-B)U_{SA} + BU_{co,s})}\Big] e^{-t^d/\tau^d} .$$

OR

$$T_s^d(t^d) = \frac{(1-B)(1-F)\,Q_T\,(1-e^{-t^d/\tau^d})}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} + \frac{(1-B)\,U_{SA}\,T_c\,(1-e^{-t^d/\tau^d})}{(1-B)U_{SA} + BU_{co,s}} +$$

$$\frac{B\cdot U_{co,s}\,T_{co}^d\,(1-e^{-t^d/\tau^d})}{(1-B)U_{SA} + BU_{co,s}} \quad \frac{(1-F_c)\,Q_T\,B\,(1-e^{-t^d/\tau^d})}{\Delta t^d((1-B)U_{SA} + BU_{co,s})} +$$

$$T_s(0)\,e^{-t^d/\tau^d} .$$

Henceforth $\phi = (1-B)U_{SA} + BU_{co,s}$. (4a) add and subtract $\frac{B \cdot U_{co,s} T_c (1 - e^{-t^d/\tau^d})}{(1-B) U_{sA} + B U_{co,s}}$ on RHS

$$\rightarrow T_s^d(t^d) = \left[\frac{(1-B)(1-F) Q_T (1 - e^{-t^d/\tau^d})}{\Delta t^d \ \phi} \right] - \left[\frac{B \cdot U_{co,s} (1 - e^{-t^d/\tau^d})(T_c - T_{co}^d)}{\phi} \right] +$$

(1st term) (use subtracted part & term 3)

$$T_c (1 - e^{-t^d/\tau^d}) + \frac{(1-F_c) Q_T B (1 - e^{-t^d/\tau^d})}{\Delta t^d \ \phi} +$$

(added part + term 2) (term 4)

$$T_s(0) e^{-t^d/\tau^d} \quad . \qquad (5)$$

(term 5)

Taking T_c to LHS of eqn (5) and combining remaining portion of term 3 with term 5

$$\rightarrow T_s^d(t^d) - T_c = \left[\frac{(1-B)(1-F) Q_T (1 - e^{-t^d/\tau^d})}{\Delta t^d \ \phi} \right] +$$

$$\left[\frac{(1-F_c) Q_T B (1 - e^{-t^d/\tau^d})}{\Delta t^d \ \phi} \right] -$$

$$\left[\frac{B U_{co,s} (1 - e^{-t^d/\tau^d})(T_c - T_{co}^d)}{\phi} \right] +$$

$$(T_s(0) - T_c) e^{-t^d/\tau^d} \quad . \qquad (6)$$

Term 1 and 2 of eqn (6) can be further combined to yield a term in the coefficient, η

$$\eta = (1-B)(1-F) + B(1-F_c)$$

OR

$$\eta = 1 - F - \cancel{B} + BF + \cancel{B} - BF_c$$

$$\eta = 1 - F + BF - BF_c \quad . \qquad (7)$$

This is the solution employed in Appendix (9).

A5

CONSERVATORY $ NO NIGHT TIME

INSULATION.

The solution of equation 1.7.8 for heat balance at night time with no insulation is considered below.

The Differential eqn is of the form

$$D\frac{dy}{dx} = -(1-B)U_{SA}(y - T_c) - B(U_{co,s}(y - T_a^n)) \quad \downarrow \simeq T_{co}^n$$

where $y = T_s^n$, $x = t^n$

$$D\frac{dy}{dx} = -A(y-C) - E(y-F)$$

$$D\frac{dy}{dx} = -(A+E)y + (AC + EF)$$

$$B^* = A + E = (1-B)U_{SA} + BU_{co,s} = \phi$$

$$\frac{dy}{dx} + \frac{B^*}{D}y = \frac{K}{D} \qquad K = AC + EF = (1-B)U_{SA}T_c + BU_{co,s}T_a^n$$

Soln.

$$y = \frac{K}{B^*} + c^* e^{-\frac{B^*}{D}x} \cdot (1) \qquad \frac{K}{B^*} = \frac{(1-B)U_{SA}T_c}{\phi} + \frac{B \cdot U_{co,s}T_a^n}{\phi}$$

$$\frac{B^*}{D} = \frac{\phi}{(mC)_s}$$

$$\therefore T_s^n(t^n) = \frac{(1-B)U_{SA}T_c}{\phi} + \frac{BU_{co,s}T_a^n}{\phi} + c^* e^{-t^n/\tau^n}. \quad (2)$$

putting $T_s^n(t^n) = T_s^d(\Delta t^d)$ at $t^n = 0$ and $(mC)_s/\phi = \tau^n$ gives,

$$T_s^d(\Delta t^d) - \frac{(1-B)U_{SA}T_c}{\phi} - \frac{BU_{co,s}T_a^n}{\phi} = c^*. \quad (3)$$

(3) $\underset{\rightarrow}{\text{in}}$ (2)

$$T_s^n(t^n) = \left[\frac{(1-B)U_{SA}\cdot T_c}{\phi} + \frac{BU_{co,s}T_a^n}{\phi}\right] + \left[T_s^d(\Delta t^d) - \frac{(1-B)U_{SA}T_c}{\phi} - \frac{BU_{co,s}T_a^n}{\phi}\right]e^{-t^n/\tau^n}.$$

$$T_s^n(t^n) = \frac{(1-B)U_{SA}T_c(1 - e^{-t^n/\tau^n})}{\phi} + \frac{BU_{co,s}T_a^n(1 - e^{-t^n/\tau^n})}{\phi} + T_s^d(\Delta t^d)e^{-t^n/\tau^n}.$$

add and subtract $\dfrac{B\cdot U_{co,s}\cdot T_c\,(1-e^{-t^n/\tau^n})}{\phi}$ to RHS

$$T_s^n(t^n) = -\left[\frac{BU_{co,s}\,(1-e^{-t^n/\tau^n})\,(T_c - T_a^n)}{\phi}\right] +$$

(subtracted part & term 2)

$$T_c\,(1-e^{-t^n/\tau^n}) +$$

(added part and term 1)

$$T_s^d(\Delta t^d)\, e^{-t^n/\tau^n} \, . \qquad (4)$$

Taking T_c to LHS of eqn (4) and combining remaining portion of term 2 with term 3

$$\longrightarrow T_s^n(t^n) - T_c = -\left[\frac{BU_{co,s}\,(1-e^{-t^n/\tau^n})\,(T_c - \overset{T_{co}^n}{\overset{\downarrow}{T_a^n}})}{\phi}\right] + (T_s^d(\Delta t^d) - T_c)\, e^{-t^n/\tau^n} \, .$$

where

$$\tau^n = (mC)_s / \phi \quad \text{and}$$

$$\phi = (1-B)\,U_{SA} + B\,U_{co,s}$$

A6

<u>Appendix 6</u>

DIRECT GAIN · EQN (A1·10)

To obtain the energy collected by room air during daytime, the integral below is evaluated as

$$Q_d^{(1)} = FQ_T + \int_0^{\Delta t^d} U_{SA}\left(T_s^d(t^d) - T_c\right)dt^d$$

$$= FQ_T + U_{SA}\int_0^{\Delta t^d}\left[\frac{(1-F)\,Q_T\,(1-e^{-t^d/\tau})}{U_{SA}\,\Delta t^d}\right]dt^d \;+$$

$$\int_0^{\Delta t^d} U_{SA}\left(T_s(0) - T_c\right)e^{-t^d/\tau}\,dt^d . \qquad (1)$$

$$= FQ_T + \left[\frac{(1-F)\,Q_T}{\Delta t^d}\left(t^d - \frac{e^{-t^d/\tau}}{-\frac{1}{\tau}}\right)\right]_0^{\Delta t^d} + U_{SA}\left[\left[T_s(0) - T_c\right]\frac{e^{-t^d/\tau}}{-\frac{1}{\tau}}\right]_0^{\Delta t^d} . \qquad (2)$$

$$= FQ_T + \left[\frac{(1-F)\,Q_T}{\cancel{\Delta t^d}}\left(\cancel{\Delta t^d} - \frac{e^{-\Delta t^d/\tau}}{-\frac{1}{\tau}}\right) + U_{SA}\left(T_s(0) - T_c\right)\frac{e^{-\Delta t^d/\tau}}{-\frac{1}{\tau}}\right] -$$

$$\left[\frac{(1-F)\,Q_T}{\Delta t^d}(0 + \tau) + U_{SA}\left(T_s(0) - T_c\right) - \tau\right] . \qquad (3)$$

$$= \cancel{FQ_T} + \left[\cancel{(1-F)Q_T} + \frac{(1-F)\cancel{Q_T}\,\tau\, e^{-\Delta t^d/\tau}}{\cancel{\Delta t^d}} - U_{SA}\left(T_s(0) - T_c\right)\tau e^{-\Delta t^d/\tau}\right] -$$

$$\left[\frac{(1-\cancel{F)Q_T}\,\tau}{\cancel{\Delta t^d}} - \tau\, U_{SA}\left(T_s(0) - T_c\right)\right] . \qquad (4)$$

$$\therefore\; Q_d^{(1)} = Q_T\left[1 - \left[\frac{(1-F)\,\tau\,(1-e^{-\Delta t^d/\tau})}{\Delta t^d}\right]\right] +$$

$$U_{SA}\left(T_s(0) - T_c\right)\tau\left(1 - e^{-\Delta t^d/\tau}\right) . \qquad (5)$$

A7 **Appendix 7**

DIRECT GAIN

To obtain the energy collected by room air during night time, the integral below is evaluated as,

$$Q_n^{(1)} = \int_0^{\Delta t^n} U_{SA}(T_s^n(t^n) - T_c)\, dt^n$$

$$= U_{SA} \int_0^{\Delta t^n} \left[\frac{(1-F)\, Q_T\, (1 - e^{-\Delta t^d/\tau})}{U_{SA}\, \Delta t^d} \right] e^{-t^n/\tau}\, dt^n \; +$$

$$U_{SA} \int_0^{\Delta t^n} \left[(T_s(0) - T_c)\, e^{-\Delta t^d/\tau} \right] e^{-t^n/\tau}\, dt^n$$

$$= \left[\frac{(1-F)\, Q_T\, (1 - e^{-\Delta t^d/\tau})\, e^{-t^n/\tau}}{\Delta t^d \quad -\frac{1}{\tau}} \right]_0^{\Delta t^n} \; +$$

$$U_{SA} \left[(T_s(0) - T_c)\, e^{-\Delta t^d/\tau} \cdot \frac{e^{-t^n/\tau}}{-\frac{1}{\tau}} \right]_0^{\Delta t^n}$$

$$\therefore Q_n^{(1)} = \left[\frac{(1-F)\, Q_T\, (1 - e^{-\Delta t^d/\tau}) \cdot -\tau\, e^{-\Delta t^n/\tau}}{\Delta t^d} \right] + U_{SA}\left[(T_s(0) - T_c)\, e^{-\frac{\Delta t^d}{\tau}} \cdot -\tau \cdot e^{-\frac{\Delta t^n}{\tau}} \right] -$$

$$\left[\frac{(1-F)\, Q_T\, (1 - e^{-\Delta t^d/\tau}) \cdot -\tau}{\Delta t^d} \right] + U_{SA}\left[(T_s(0) - T_c)\, e^{-\frac{\Delta t^d}{\tau}} \cdot -\tau \right]$$

$$\therefore Q_n^{(1)} = \left[\frac{(1-F)\, G_T\, \tau\, (1 - e^{-\Delta t^d/\tau})(1 - e^{-\Delta t^n/\tau})}{\Delta t^d} \right] +$$

$$U_{SA}\left[(T_s(0) - T_c)\, e^{-\Delta t^d/\tau} \cdot \tau\, (1 - e^{-\Delta t^n/\tau}) \right].$$

Appendix 8

A8 DIRECT GAIN

The solution for the average useful daytime (night time) energy gain Q_d (Q_n), i.e. solution of the two simultaneous equations (1.3.17) and (1.3.18); eliminating the second terms respectively and writing for Q_n, ($Q_T - Q_d$). Hence,

$$Q_d = Q_T\left[1 - \frac{(1-F)\tau(1-e^{-\Delta t^d/\tau})}{\Delta t^d}\right] + U_{sA}(T_s(0) - T_c)\tau(1-e^{-\Delta t^d/\tau}) \quad (1.3.17) \quad (1)$$

$$Q_T - Q_d = \frac{Q_T(1-F)\tau(1-e^{-\Delta t^d/\tau})(1-e^{-\Delta t^n/\tau})}{\Delta t^d} + U_{sA}(T_s(0)-T_c)\tau e^{-\Delta t^d/\tau}(1-e^{-\Delta t^n/\tau}) \quad (1.3.18) \quad (2)$$

Henceforth $(1-e^{-\Delta t^d/\tau})$ and $(1-e^{-\Delta t^n/\tau})$ are ϕ_d and ϕ_n respectively.

(1.3.18) $\Rightarrow$

$$Q_d = Q_T\left[1 - \frac{(1-F)\tau\phi_d \phi_n}{\Delta t^d}\right] + U_{sA}(T_s(0)-T_c)\tau e^{-\Delta t^d/\tau}\phi_n \quad (3)$$

$\{\text{Eqn } 1 \times e^{-\Delta t^d/\tau}\phi_n\} + [\text{Eqn } 3 \times \phi_d]$ gives,

$$Q_d e^{-\Delta t^d/\tau}\phi_n = Q_T\left[e^{-\Delta t^d/\tau}\phi_n - e^{-\Delta t^d/\tau}\phi_n \frac{(1-F)\tau\phi_d}{\Delta t^d}\right] +$$

$$+ \; Q_d\phi_d \qquad Q_T\left[\phi_d - \phi_n\phi_d\frac{(1-F)\tau\phi_d}{\Delta t^d}\right] \quad (4)$$

OR

$$Q_d\left[e^{-\Delta t^d/\tau}\phi_n + \phi_d\right] = Q_T\left[e^{-\Delta t^d/\tau}\phi_n + \phi_d\right] - Q_T\cdot\frac{(1-F)\tau\phi_d}{\Delta t^d}\left[e^{-\frac{\Delta t^d}{\tau}}\phi_n + \phi_d\phi_n\right] \quad (5)$$

$\div$ both sides of eqn (5) by cft of Q_d term

$$\therefore Q_d = Q\left[1 - \left[\frac{e^{-\frac{\Delta t^d}{\tau}}\phi_n + \phi_d\phi_n}{e^{-\Delta t^d/\tau}\phi_n + \phi_d}\right]\frac{(1-F)\tau\phi_d}{\Delta t^d}\right]$$

$$\therefore Q_d = Q_T\left[1 - \left[\frac{(\cancel{e^{-\frac{\Delta t^d}{\tau}}} - \cancel{e^{-(\frac{\Delta t^d}{\tau}+\frac{\Delta t^n}{\tau})}} + 1 - e^{-\frac{\Delta t^n}{\tau}} - \cancel{e^{-\frac{\Delta t^d}{\tau}}} + e^{-(\Delta t^d+\Delta t^n)/\tau})(1-F)\tau\phi_d}{(\cancel{e^{-\Delta t^d/\tau}} - e^{-(\Delta t^d/\tau + \Delta t^n/\tau)}) + 1 - \cancel{e^{-\Delta t^d/\tau}} \quad \Delta t^d}\right]\right]$$

$$\therefore Q_d = Q_T\left[1 - \left[\frac{(1-F)\tau\phi_d\phi_n}{\Delta t^d(1-e^{-(\Delta t^d+\Delta t^n)/\tau})}\right]\right] \quad (6)$$

Q.E.D

Appendix 9

A9 DIRECT GAIN AND CONSERVATORY:

Here $\phi = (1-B)U_{SA} + BU_{co,s}$

To obtain the energy collected by room air during daytime for the direct gain system with attached conservatory, the integral below is evaluated as

The soln for $T_s^d(t^d) - T_c$ is from eqn (A4.7)

$$Q_d^{(1)} = FQ_T + \int_0^{\Delta t^d} U_{SA}(T_s^d(t^d) - T_c)\,dt^d$$

$$= FQ_T + \int_0^{\Delta t^d} U_{SA}\left[\frac{\eta Q_T(1 - e^{-t^d/\tau^d})}{\Delta t^d\,\phi}\right]dt^d -$$

$$U_{SA}\int_0^{\Delta t^d}\left[\frac{B\cdot U_{co,s}(1 - e^{-t^d/\tau^d})(T_c - T_{co}^d)}{\phi}\right]dt^d +$$

$$\int_0^{\Delta t^d} U_{SA}(T_s(0) - T_c)\,e^{-t^d/\tau^d}\,dt^d \qquad (1)$$

$$= FQ_T + U_{SA}\left[\frac{\eta Q_T\left(t^d - \frac{e^{-t^d/\tau^d}}{-1/\tau^d}\right)}{\Delta t^d\,\phi}\right]_0^{\Delta t^d} -$$

$$U_{SA}\left[\frac{B\cdot U_{co,s}\left(t^d - \frac{e^{-t^d/\tau^d}}{-1/\tau^d}\right)(T_c - T_{co}^d)}{\phi}\right]_0^{\Delta t^d} +$$

$$U_{SA}\left[(T_s(0) - T_c)\frac{e^{-t^d/\tau^d}}{-1/\tau^d}\right]_0^{\Delta t^d}$$

$$= FQ_T + \left[\frac{U_{SA}\eta Q_T\left(\Delta t^d - \frac{e^{-\Delta t^d/\tau^d}}{-1/\tau^d}\right)}{\Delta t^d\,\phi} - \frac{U_{SA}B\,U_{co,s}\left(\Delta t^d - \frac{e^{-\frac{\Delta t^d}{\tau^d}}}{-1/\tau^d}\right)(T_c - T_{co}^d)}{\phi} +\right.$$

$$\left. U_{SA}(T_s(0) - T_c)\frac{e^{-\Delta t^d/\tau^d}}{-1/\tau^d}\right] - \left[\frac{U_{SA}\eta Q_T(0 + \tau^d)}{\Delta t^d\,\phi} - \frac{U_{SA}B\,U_{co,s}(0 + \tau^d)(T_c - T_{co}^d)}{\phi}\right.$$

$$\left. - U_{SA}(T_s(0) - T_c)\tau^d\right] \qquad (2)$$

Expanding term 1 and 2 of eqn (2) gives

$$Q_d^{(1)} = FQ_T + \left\{ \frac{U_{SA}\eta Q_T}{\phi} + \frac{U_{SA}\eta Q_T \tau^d e^{-t^d/\tau^d}}{\Delta t^d \phi} - \left[\frac{U_{SA} B U_{co.s} \Delta t^d}{\phi} + \frac{U_{SA} B U_{co.s} \tau^d e^{-\Delta t^d/\tau^d}}{\phi} \right] (T_c - T_{co}^d) - U_{SA}(T_s(0) - T_c)\tau^d e^{-\Delta t^d/\tau^d} \right\} - \frac{U_{SA}\eta Q_T \tau^d}{\Delta t^d \phi} + \frac{U_{SA} B U_{co.s} \tau^d (T_c - T_{co}^d)}{\phi} + U_{SA}(T_s(0) - T_c)\tau^d . \quad (3)$$

Combining terms (1,2,6) and terms (5 and 8) respectively of eqn (3)

$$Q_d^{(1)} = FQ_T + \frac{U_{SA}\eta Q_T}{\phi}\left[1 - \left[\frac{\tau^d(1-e^{-\Delta t^d/\tau^d})}{\Delta t^d}\right]\right] + U_{SA}(T_s(0) - T_c)\tau^d(1 - e^{-\Delta t^d/\tau^d}) - L_1 . \quad (4)$$

where L_1 is a combination of terms 3, 4 and 7 of eqn (3) $\$$

$$L_1 = \frac{U_{SA} B U_{co.s} \tau^d (1 - e^{-\Delta t^d/\tau^d} - \Delta t^d/\tau^d)(T_c - T_{co}^d)}{\phi} . \quad (5)$$

L_1 represents the contributions of the conservatory in heating room air

$$\xrightarrow{(4)} \therefore Q_d^{(1)} = FQ_T\left[1 - \frac{U_{SA}\eta}{F\phi}\left[\frac{\tau^d(1-e^{-\Delta t^d/\tau^d})}{\Delta t^d} - 1\right]\right] + U_{SA}(T_s(0) - T_c)\tau^d(1 - e^{-\Delta t^d/\tau^d}) - L_1 . \quad (6)$$

Appendix 10

A10 DIRECT GAIN & CONSERVATORY (Night-time Insulation)

To evaluate the energy collected by room air at night time for the direct gain system employing night time insulation, with an attached conservatory, the integral below is evaluated as follows;

where $\phi_d^d = (1 - e^{-\Delta t^d/\tau^d})$ and $\phi_n^n = (1 - e^{-\Delta t^n/\tau^n})$

Here heat transfer at night time from storage is only to room air.

The soln for $(T_s^n(t^n) - T_c)$ was given in eqn 1.7.11

Now, $Q_n^{(1)}$ is given by

$$Q_n^{(1)} = \int_0^{\Delta t^n} u_{sA} (T_s^n(t^n) - T_c)\, dt^n \quad . \qquad (1)$$

$\overset{1.7.11}{\Rightarrow}$

$$Q_n^{(1)} = \int_0^{\Delta t^n} u_{sA} \left[\frac{Q_T \phi_d^d \eta}{\Delta t^d \phi} \right] e^{-t^n/\tau^n} dt^n + \int_0^{\Delta t^n} u_{sA} \left[(T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \right] e^{-t^n/\tau^n} dt^n - \int_0^{\Delta t^n} u_{sA} \left[\frac{B\, u_{co,s}\, \phi_d^d (T_c - T_{co}^d)}{\phi} \right] e^{-t^n/\tau^n} dt^n \qquad (2)$$

$$= \left. \frac{u_{sA} Q_T \phi_d^d \eta\, e^{-t^n/\tau^n}}{\Delta t^d \phi \cdot -1/\tau^n} \right]_0^{\Delta t^n} + \left. u_{sA} \left[(T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \right] \frac{e^{-t^n/\tau^n}}{-1/\tau^n} \right]_0^{\Delta t^n} - \left. \frac{u_{sA} B\, u_{co,s}\, \phi_d^d (T_c - T_{co}^d) e^{-t^n/\tau^n}}{\phi \cdot -1/\tau^n} \right]_0^{\Delta t^n} . \qquad (3)$$

$$= \left[\frac{u_{sA} Q_T \eta \cdot -\tau^n \phi_d^d e^{-\Delta t^n/\tau^n}}{\Delta t^d \phi} - u_{sA} \left[(T_s(0) - T_c) e^{-\frac{\Delta t^d}{\tau^d}} \cdot e^{-\frac{\Delta t^n}{\tau^n}} \cdot \tau^n \right] + \frac{u_{sA} B\, u_{co,s}\, \tau^n \phi_d^d e^{-\Delta t^n/\tau^n} (T_c - T_{co}^d)}{\phi} \right] + \left[\frac{u_{sA} Q_T \tau^n \eta\, \phi_d^d}{\Delta t^d \phi} + u_{sA} \left[(T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \cdot \tau^n \right] - \frac{u_{sA} B\, u_{co,s}\, \tau^n \phi_d^d (T_c - T_{co}^d)}{\phi} \right] . \qquad (4)$$

combining similar terms in eqn. (4) yields

$$Q_n^{(1)} = \frac{\eta Q_T U_{SA} \tau^n \phi_d^d \phi_n^n}{\Delta t^d \phi} + U_{SA} \tau^n (T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \phi_n^n - L_2 \quad (5)$$

where

$$L_2 = \frac{U_{SA} B U_{co,s} \tau^n \phi_d^d \phi_n^n (T_c - T_{co}^d)}{\phi} \quad (6)$$

or

$$Q_n^{(1)} = \frac{m Q_T F U_{SA} \tau^n \phi_d^d \phi_n^n}{\Delta t^d \phi} + U_{SA} \tau^n (T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \cdot \phi_n - L_2 \quad (7)$$

where

$$m = \eta / F \quad (8)$$

and

$$\phi = (1-B) U_{SA} + B U_{co,s}$$

Appendix 11

A11 DIRECT GAIN AND CONSERVATORY (No night insulation)

To evaluate the energy collected by room air at night time for the direct gain system employing NO night time insulation, with attached conservatory, the eqn. (1.7.14) is substituted in (1.7.16) and the integral evaluated as follows, where τ^d and τ^n are given by eqn. (1.7.4) and (1.7.13) respectively. The energy will be distinguished by an asterisk (*)

$$Q_n^{(1*)} = \int_0^{\Delta t^n} u_{sA} \left(T_s^n(t^n) - T_c \right) dt^n \qquad (1)$$

$$= \int_0^{\Delta t^n} u_{sA} \left[\frac{Q_T \phi_d^d \eta \, e^{-t^n/\tau^n}}{\Delta t^d \, \phi} \right] dt^n +$$

$$\int_0^{\Delta t^n} \left[(T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \right] e^{-t^n/\tau^n} dt^n -$$

$$\int_0^{\Delta t^n} u_{sA} \left[\frac{B \cdot u_{co,s} \, \phi_d^d (T_c - T_{co}^d)}{\phi} \right] e^{-t^n/\tau^n} dt^n -$$

$$\int_0^{\Delta t^n} u_{sA} \left[\frac{B \, u_{co,s} (1 - e^{-t^n/\tau^n})(T_c - T_{co}^n)}{\phi} \right] dt^n . \qquad (2)$$

The first three terms of the Integral have just been evaluated. See Appendix 10. We proceed by evaluating term 4

$$\int_0^{\Delta t^n} u_{sA} \left[\frac{B \, u_{co,s} (1 - e^{-t^n/\tau^n}) (T_c - T_{co}^n)}{\phi} \right] dt^n =$$

$$\left[\frac{u_{sA} B \, u_{co,s} \left(t^n - \frac{e^{-t^n/\tau^n}}{-1/\tau^n} \right) (T_c - T_{co}^n)}{\phi} \right]_0^{\Delta t^n} =$$

$$\left[\frac{u_{sA} B \, u_{co,s} (\Delta t^n + \tau^n e^{-\Delta t^n/\tau^n})(T_c - T_{co}^n)}{\phi} \right] -$$

$$\left[\frac{u_{sA} B \, u_{co,s} (0 + \tau^n) \, (T_c - T_{co}^n)}{\phi} \right] . \qquad (3)$$

$$= \frac{U_{SA}\, B\, U_{co.s}\, \Delta t^{n} (T_c - T_{co}^{n})}{\phi} + \frac{U_{SA}\, B\, U_{co.s}\, \tau^{n} e^{-\Delta t^{n}/\tau^{n}} (T_c - T_{co}^{n})}{\phi} - \frac{U_{SA}\, B\, U_{co.s}\, \tau^{n} (T_c - T_{co}^{n})}{\phi} .$$

$$\text{Term } 4 = \frac{U_{SA}\, B\, U_{co.s}\, \tau^{n} (\Delta t^{n}/\tau^{n} + e^{-\Delta t^{n}/\tau^{n}} - 1)(T_c - T_{co}^{n})}{\phi} .$$

Writing the solution for the first three terms, (see Appendix 10) together with the integral of term 4 yields the energy collected by room air at night time for a direct gain system employing NO night time insulation, with attached conservatory as

$$Q_n^{(1*)} = Q_n^{(1)} - L_3 . \quad (4)$$

where $Q_n^{(1)}$ is the energy room air collects at night time for a system employing night time insulation (see Appendix 10 and eqn. (1.7.18); and L_3 is given by

$$L_3 = \frac{U_{SA}\, B\, U_{co.s}\, \tau^{n} (\Delta t^{n}/\tau^{n} + e^{-\Delta t^{n}/\tau^{n}} - 1)(T_c - T_{co}^{n})}{\phi} . \quad (5)$$

Hence the effect of not employing night time insulation is to reduce the energy collected by room air by a value equal to L_3 .

where ϕ and ϕ_d^d are as defined in Appendix 10 .

Appendix 12

A12 DIRECT GAIN AND CONSERVATORY

The solution for the average useful daytime (night time) gain Q_d (Q_n) is obtained by eliminating the term in $T_s(0) - T_c$ and L_1, L_2 between equation (1.7.17) and (1.7.18) and writing for Q_n ; $Q_T - Q_d$ in eqn (1.7.18).

Night time Insulation

$$Q_d = FQ_T\left[1 - \frac{m\,U_{SA}}{\phi}\left[\frac{\tau^d \phi_d^d}{\Delta t^d} - 1\right]\right] + U_{SA}(T_s(0) - T_c)\tau^d \phi_d^d + L_1 \qquad (1a)$$

A9.6 OR (1.7.17)

$$Q_T - Q_d = \frac{mFQ_T U_{SA} \tau^n \phi_d^d \phi_n^n}{\Delta t^d \phi} + U_{SA}\tau^n (T_s(0) - T_c) e^{-\Delta t^d/\tau^d} \phi_n^n - L_2 \qquad (2a)$$

A10.7 OR (1.7.18)

We re-arrange eqn. (1.7.17)

$$Q_d = Q_T\left[F - \frac{mFU_{SA}}{\phi}\left[\frac{\tau^d \phi_d^d}{\Delta t^d} - 1\right]\right] + (\text{term } 2) + L_1 \qquad (1)$$

Re-arrange eqn. (1.7.18)

$$Q_d = Q_T\left[1 - \frac{mF\,U_{SA}\,\tau^n\,\phi_d^d\,\phi_n^n}{\Delta t^d\,\phi}\right] - (\text{term } 2) + L_2 \qquad (2)$$

$\{\text{Eqn }(1) \times [\tau^n e^{-\Delta t^d/\tau^d} \phi_n^n]\} + \{\text{Eqn } 2 \times [\tau^d \phi_d^d]\}$ gives,

$$\therefore Q_d[\tau^n e^{-\Delta t^d/\tau^d} \phi_n^n] + Q_d[\tau^d \phi_d^d] = Q_T\left[F\tau^n e^{-\Delta t^d/\tau^d} \phi_n^n - \frac{mFU_{SA}\tau^n e^{-\frac{\Delta t^d}{\tau^d}} \phi_n^n}{\Delta t^d\,\phi}\left[\tau^d \phi_d^d - \Delta t^d\right]\right] + L_1 \tau^n e^{-\Delta t^d/\tau^d} \phi_n^n + Q_T\left[\tau^d \phi_d^d - \frac{mFU_{SA}\,\phi_n^n \tau^d \phi_d^d \tau^n \phi_d^d}{\Delta t^d\,\phi}\right] + L_2 \tau^d \phi_d^d . \qquad (3a)$$

$$Q_d(\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d) = Q_T\Big\{(F\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d) - \frac{mFU_{sA}[\tau^n \overset{*}{\phi_d^d} - \Delta t^d e^{-\Delta t^d/\tau^d}]\phi_n^n}{\Delta t^d\phi}\Big\} + L_1\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + L_2\tau^d\phi_d^d \qquad (A12.3)$$

* See how added in (A15)

where L_1 and L_2 is as defined in eqns 1.7.36 and 1.7.37. From these eqns the relationship between L_1 and L_2 is

$$\left|\frac{L_1}{L_2}\right| = \frac{\tau^d(1-e^{-\Delta t^d/\tau^d} - \Delta t^d/\tau^d)}{\tau^n\phi_d^d\phi_n^n} \qquad (A12.4)a$$

add and subtract $\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n$ in the numerator of eqn. (A12.3), term 1 (R.H.S); divide both sides by coefficient of Q_d term

$$\therefore Q_d = Q_T\Big[1 - (1-F)\frac{\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n}{\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d} - \frac{mFU_{sA}\overset{*}{\phi_n^n}[\tau^n[\tau^d\phi_d^d - \Delta t^d e^{-\Delta t^d/\tau^d}]]}{\Delta t^d\phi[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d]} + \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + L_2\tau^d\phi_d^d}{Q_T[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d]}\Big] \qquad (A12.4)$$

This is the exact solution for Q_d .

Since $Q_n = Q_T - Q_d$, it follows from equation (A12.4) ; that

$$Q_n = Q_T \left[\frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d} \phi_n^n}{\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d \phi_d^d} + \frac{mFU_{sA}\phi_n^n[\tau^n[\tau^d\phi_d^d - \Delta t^d e^{-\Delta t^d/\tau^d}]]}{\Delta t^d \phi[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d \phi_d^d]} \right.$$
$$\left. - \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + L_2\tau^d\phi_d^d}{Q_T[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d]} \right] \quad (A12.5)$$

Hence, from eqns A12.4 and A12.5

$$C_o^d = \left(\frac{\bar{L}}{\bar{L}_d}\right)\left[1 - \frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n}{\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d} - \frac{mFU_{sA}\tau^n\phi_n^n[\tau^d\phi_d^d - \Delta t^d e^{-\Delta t^d/\tau^d}]}{\Delta t^d\phi[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d]} + \right.$$
$$\left. \frac{L_1\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + L_2\tau^d\phi_d^d}{Q_T[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\phi_d^d]} \right] \quad (A12.6)$$

and

$$C_o^n = \left(\frac{\bar{L}}{\bar{L}_n}\right)\left[\frac{(1-F)\tau^n e^{-\Delta t^d/\tau^d}\ \phi_n^n}{\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d\ \phi_d^d} + \frac{mFU_{SA}\ \phi_n^n \tau^n[\tau^d \phi_d^d - \Delta t^d e^{\frac{-\Delta t^d}{\tau^d}}]}{\Delta t^d\ \phi\ [\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d \phi_d^d]} - \frac{L_1 \tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + L_2 \tau^d \phi_d^d}{Q_T[\tau^n e^{-\Delta t^d/\tau^d}\phi_n^n + \tau^d \phi_d^d]}\right] \qquad (A12.7)$$

where Q_T per day = Q_T (J/m^2) over heating season, divided by 120 days, the length of the season and

$$\tau^d = (mC)_s/\phi \qquad (A12.8)$$

$$\tau^n = (mC)_s/(1-B)U_{SA} \qquad (A12.9)$$

$$m = \eta/F = (1-F+BF-BF_c)/F \qquad A(12.10)$$

Strictly speaking m should be $m = S_A(1-F+BF-BF_c)/F$ where S_A is the mass surface absorbance of storage e.g. S_A = 0.8 (1,2).

It is shown in equations 1.8.8 to 1.8.13 that the values of C_o^d and C_o^n for a direct gain system using a conservatory eqns A12.6 and A12.7 above become identical and reduce to those presented by Gordon and Zarmi (1,2), for a direct gain system only, i.e. no attached conservatory, see eqns 1.3.24 and 1.3.25.

L_1 and L_2 are defined is appendices 9 and 10 as

$$L_1 = \frac{U_{SA} U_{co,s} B \tau^d (1 - e^{-\Delta t^d/\tau^d} - \Delta t^d/\tau^d)(T_c - T_{co}^d) \times 3600}{\phi} \quad (A12.11)$$

$$L_2 = \frac{U_{SA} U_{co,s} B \tau^n \phi_d^d \phi_n^n (T_c - T_{co}^d) \times 3600}{\phi} \quad (A12.12)$$

CONTRIBUTIONS BY CONSERVATORY

(a) Interpretation of L_1 and L_2 Values $[J/m^2]$

Values of L_2 as deined are (-ve). From eqn. A12.5 we see that this represents an addition to Q_n. L_2 is the useful gain from conservatory collected at daytime put to use at night time. $L_2 = f(T_{co}^d)$ for the house in (A12.12). From equation A12.4 the useful gain at daytime Q_d for the house is reduced by the useful gain that is put to use at night time, $f(L_2)$, and added by the useful gain from conservatory that is put to use at daytime $f(L_1)$

Eqn. A12.4 has 3 types of terms.

Term 1. A function of $(1-F)$

Term 2. A function of storage properties

Term 3. A function of L_1, L_2, L_3 (no night insulation)

* Therefore the contributions by the conservatory at daytime in the house heating load is the term in term 3 with L_1, and at night time is the term in term 3 with L_2. Hence from eqns. A12.4 and A12.5 the contributions by the conservatory to the house heating load at both daytime and night time can be written as

$$Q_{cc} = Q_{conser,contr.} = \frac{L_1 \tau^n e^{-\Delta t^d/\tau^d} \phi_n^n + L_2 \tau^d \phi_d^d}{\tau^n e^{-\Delta t^d/\tau^d} \phi_n^n + \tau^d \phi_d^d} \quad (A\,12.13)$$

Term 3 (eqn.A12.4) is conservatory contribution,absorbed by storage strictly speaking.

In the case of no night insulation, L_2^* simply becomes (L_2 + L_3). Since L_3 is -ve , inspection of eqn A12.5 shows that L_3 is subtracted from Q_n and f(L_3) term in eqn. A12.5 represents the night time losses due to no night insulation. Therefore for a system with attached conservatory and no night insulation

$$Q_{cc}^* = \frac{L_1 \tau^n e^{-\Delta t^d/\tau^d} \phi_n^n + (L_2 + L_3)\tau^d \phi_d^d}{\tau^n e^{-\Delta t^d/\tau^d} \phi_n^n + \tau^d \phi_d^d} \quad (A\,12.14)$$

The (L_2+L_3) term being heat collected by conservatory at daytime and stored for use at night time (A10) plus night losses due to no night insulation (A 11)

mf in term 3 of eqn. A12.4 is given by

$$\begin{aligned} mf &= 1-(F_{eq}) \\ &= 1-((1-B)F + BF_c) \\ &= 1-(F - BF + BF_c) \\ &= 1 - F + BF - BF_c \quad (A\,12.15) \end{aligned}$$

Therefore we see that the contributions by the conservatory for a system with and without night insulation is a

$$f(\tau^d, \tau^n, \Delta t^d, L_1(T_c - T_{co}^d), L_2(T_c - T_{co}^d)$$
$$L_3(T_c - T_{co}^n) \equiv (T_c - T_{co}^d)K^*, F, B, F_c)$$

K^* being a constant, assuming various values.

From eqn (A 12.15).

if we assume $F = F_c$ as may be the case

$$mF = (1-F)$$

F_c = fraction of transmitted insolation into conservatory not absorbed by storage.

F = fraction of transmitted insolation into room not absorbed by storage.

Generally, F may not equal F_c as the furniture and utensils in the room absorb more, making $F \ll F_c$.

In Appendix 12,

$\phi_d^d = (1 - e^{-\Delta t^d/\tau^d})$, $\phi_n^n = (1 - e^{-\Delta t^n/\tau^n})$, and

$$\phi = (1-B)U_{SA} + B\,U_{co,s} \qquad (A12.16)$$

APPENDIX 13

Q_d (Q_n) for no night insulation and conservatory

In order to evaluate the average daytime (night time) useful energy gain for the heating season Q_d^*(Q_n^*) for the case with no night time insulation, we need to express ($T_c - T_{co}^d$) in terms of ($T_c - T_{co}^n$).

In the work by Gordon and Zarmi (1), by using a backup source to keep room temperrature at T_c, they did not distinguish between the room temperatures at daytime and night time. See their eqns (20) and (21). In keeping with such assumptions, if we assumed that the conservatory is kept at a uniform temperature T_{co} during daytime and night time, and backup is used to keep this temperature constant, so that $T_{co}^d = T_{co}^n = T_{co}$ it would not be reasonable in practice.

Hence we rather assume or introduce a constant term K where K for a given day or (all average days thereof) in the heating season is

$$K = (T_c - T_{co}^d) / (T_c - T_{co}^n) \qquad (*)$$

Typical values of K may lie in the range

$$K = \frac{18{\cdot}3 - 14{\cdot}5}{18{\cdot}3 - (0 \rightarrow 11^\circ C)} \quad [\text{ref } 2] T_c$$

Or

$$0{\cdot}2 < K < 0{\cdot}5 \qquad (1)$$

K is a climatic factor and is given for any day in the heating season.

Hence

$$(T_c - T_{co}^d) = K(T_c - T_{co}^n) \qquad (2)$$

and the average value relationship can be known for any typical day.

Hence we can write the relationship for L_3 as

$$L_3 = \frac{-U_{SA}\, B\, U_{co,s}\, \tau^n (1 - e^{-\Delta t^n/\tau^n} - \Delta t^n/\tau^n)(T_c - T_{co}^d)}{K[(1-B)U_{SA} + BU_{co,s}]} \qquad (3)$$

We can then study how the fractional temperature difference between room air and conservatory ($T_c - T_{co}$) at daytime and night time affects the SHF.

In other words how $K = (T_c - T_{co}^d)/(T_c - T_{co}^n)$ affects SHF,

Clearly, low values of K (e.g. 0.2) characterise cold night days e.g. ($T_{co}^n = 0°C$); and large values of K (e.g. 0.5) characterise warm nights (e.g. $T_{co}^n = 11°C$).

In eqn (*) above we have assumed backup is used to keep room air temperature at a constant value T_c (e.g. 18.3°C or 65°F) see ref.2 pg 344. Hence typical K values lie in the range ($0.2 < K < 0.5$)

Also -ve values of K represent days in the heating season when the temperature in the conservatory during daytime T_{co}^d is higher than the room average temperature T_c . Hence, from eqn. (*) we summarise as follows,

INTERPRETATION OF K- VALUES

0°C

T_{co}^d T_{co}^n	Influence on K of T_{co}^d		Influence on K of T_{co}^n	
K	Large -ve K e.g K=-0.5	Low -ve K e.g. K=-0.2	Low K +ve e.g K = 0.2	High +ve K e.g K=0.5
T_{co}^d T_{co}^n K	$T_{co}^d >> T_c$ e.g T_{co}^d = 22.1°C+	$T_{co}^d > T_c$ e.g. T_{co}^d = 22.1°C	T_{co}^n Low e.g 0°C T_{co}^d Low e.g 14.5°C 0°C	T_{co}^n high e.g 11°C T_{co}^d Low e.g 14.5°C
	-ve K	values	+ve K	values
	T_{co}^n high e.g 11°C REGION 1	T_{co}^n low e.g 0°C REGION 2	REGION 3	REGION 4

0°C

A typical value or K to be used for heating season computations when T_{co}^n is high e.g. 11°C and T_{co}^d is higher than T_c (e.g. 22.1°C) is from eqn(*) above.

K = - 0.52

1. Hence we summarise that K is defined such that for nights with T_{co}^{n} high (e.g. 11°C) and T_{co}^{d} (high e.g. 22.1°C); K values are high and -ve e.g. - 0.5.

2. For nights with low T_{co}^{n} and high T_{co}^{d} (e.g. 22.1°C) K is low and -ve . $T_{co}^{n} = 0°C$.

   e.g. K = -0.2.

3. For nights with low T_{co}^{n} (e.g. $T_{co}^{n} = 0°C$) it is reasonable to assume T_{co}^{d} is lower than T_c since backup is used to keep room temperature at T_c. $\therefore$ $T_{co}^{d} < T_c$. or for nights with low T_{co}^{n} (e.g. 0 C) and low $T_{co}^{d} < T_c$ e.g (14.5°C) (K is low and +ve e.g. K = 0.2).

4. For nights with high T_{co}^{n} and low T_{co}^{d} (14.5 °C) (K = 0.5). ($T_{co}^{n} = 11°C$) .

Warm Days

A typical region for the heating season is region 1 of the above figure which gives a typical value of K as

$$K = - 0.52$$

using $T_{co}^{d} = 22.1$, $T_c = 18.3°C$, and $T_{co}^{n} = 11°C$.

Still much warmer days

If we increase T_{co}^{d} and T_{co}^{n} e.g.

$T_{co}^{d} = 25.1^{\circ}C$ and $T_{co}^{n} = 14^{\circ}C$ we note from eqn. *

$$K = -1.6$$

Summary

Hence, typical heating season values of K will vary from

$$K = -0.52 \quad \text{to} \quad K = -1.6$$

(warm days)

$T_{co}^{d} = 22.1^{\circ}C$

$T_{co}^{n} = 11^{\circ}C$

(hot days)

$T_{co}^{d} = 25.1^{\circ}C$

$T_{co}^{n} = 14^{\circ}C$

Cold days and nights values

cold nights/days

$T_{co}^{d} = 14.5^{\circ}C$

$T_{co}^{n} = 0^{\circ}C$

$K = 0.2$

snow nights/days

$T_{co}^{d} = 0^{\circ}C$

$T_{co}^{n} = -5^{\circ}C$

$K = 0.78$

Typical values of K are therefore for ($T_c = 18{\cdot}3°C$)

$K = 0$

$K = -1{\cdot}6$	$K = -0{\cdot}52$	$K = 0{\cdot}2$	$K = 0{\cdot}78$
$T_{co}^{d} = 25{\cdot}1°C$	$T_{co}^{d} = 22{\cdot}1°C$	$T_{co}^{d} = 14{\cdot}5°C$	$T_{co}^{d} = 0°C$
$T_{co}^{n} = 14°C$	$T_{co}^{n} = 11°C$	$T_{co}^{n} = 0°C$	$T_{co}^{n} = -5°C$
(hottest)	(hot)	(cold)	(snow)

Warmer days and nights | colder days and nights

$\longleftarrow$ -ve | +ve $\longrightarrow$

In our computations we therefore assume the reference house as in (2) $T_c = 18{\cdot}3°C$ with the further assumptions $T_{co}^{d} = 25{\cdot}1°C$ and $T_{co}^{n} = 14°C$. For this reference house $K = -1.6$.

We shall now return to the solution of $Q_d^{(1*)}$ using K such that

$$K^{*} = 1/K = (T_c - T_{co}^{n})/(T_c - T_{co}^{d}) \qquad (4)$$

Hence $(T_c - T_{co}^{n}) = K^{*}\,(T_c - T_{co}^{d})$.

Typical values of K* are therefore (where K* is a climatic factor representing the daytime and night time conditions in the conservatory),

-ve $\longleftarrow$ K^{*} = 0 $\longrightarrow$ +ve

Colder days and nights in conservatory. | warmer days and nights in conservatory.

$$-2{\cdot}0 \leqslant K^{*} \leqslant 5 \qquad (5)$$

APPENDIX 14

For the system with conservatory and NO night insulation, the daytime useful energy gain is $Q_d^{(1)}$, the same as for the case with night insulation but the night time is different (see Appendix 11) from the case where night insulation is used. Hence,

$$Q_d = Q_d^{(1*)} = Q_d^{(1)} = FQ_T\left[1 - \frac{mU_{SA}}{\phi}\left[\frac{\tau^d \phi_d^d}{\Delta t^d} - 1\right]\right]$$
$$+ U_{SA}(T_s(0) - T_c)\tau^d \phi_d^d + L_1 \qquad (1)$$

and from Appendix 11

$$Q_n^{1*} = Q_T - Q_d = \frac{mFQ_T U_{SA}\tau^n \phi_d^d \phi_n^n}{\Delta t^d \ \phi} + U_{SA}\tau^n (T_s(0) - T_c)e^{-\Delta t^d/\tau^d}\phi_n^n -$$
$$(L_2 + L_3) \qquad (2)$$

where from eqn.(1.7.4) and (1.7.13), $\tau^d = \tau^n = (mC)_s/\phi$ (3)

different from case with night insulation

From previous sections,

$$L_1 = \frac{U_{SA} B U_{co,s} \tau^d (1 - e^{-\Delta t^d/\tau^d} - \Delta t^d/\tau^d)(T_c - T_{co}^d)}{\phi} \qquad (4)$$

$$L_2 = \frac{U_{SA} B U_{co,s} \tau^n \phi_d^d \phi_n^n (T_c - T_{co}^d)}{\phi} \qquad (5)$$

$$L_3 = \frac{-U_{SA} B U_{co,s} \tau^n (1 - e^{-\Delta t^n/\tau^n} - \Delta t^n/\tau^n) K^* (T_c - T_{co}^d)}{\phi} \qquad (6)$$

where $\phi_d^d = (1 - e^{-\Delta t^d/\tau^d})$, $\phi_n^n = (1 - e^{-\Delta t^n/\tau^n})$ \$

$\phi = (1 - B)U_{SA} + BU_{co,s}$. (7)

where K* in the eqn for L_3 is given by

$K^* = (T_c - T_{co}^{n})/(T_c - T_{co}^{d})$ and typical values

are presented,

in Appendix 13.

We proceed by eliminating the terms in $(T_s(0) - T_c)$; between eqn (1) and (2) above.

The solution will be basically the same as in Appendix 12 (night insulation) with L_2 replaced by $(L_2 + L_3)$.

APPENDIX 15

Here we show how the coefficient of the terms in $\dfrac{mFU_{SA}(1-e^{-\Delta t^n/\tau^n})}{\Delta t^d((1-B)U_{SA}+BU_{co,s})}$ of eqn (A12·3a) was arrived at in Eqn (A12·3) and (A12·4).

The coefficient referred as NOM2 is

$$
\begin{aligned}
\text{NOM2} &= \tau^n\tau^d e^{-\Delta t^d/\tau^d}(1-e^{-\Delta t^d/\tau^d}) - \tau^n e^{-\Delta t^d/\tau^d}\cdot\Delta t^d + \\
&\quad \tau^n\tau^d(1-e^{-\Delta t^d/\tau^d})(1-e^{-\Delta t^d/\tau^d}) \\
&= \tau^d\tau^n(1-e^{-\Delta t^d/\tau^d})(\cancel{e^{-\Delta t^d/\tau^d}} + 1 - \cancel{e^{-\Delta t^d/\tau^d}}) - \tau^n\Delta t^d e^{-\Delta t^d/\tau^d} \\
&= \tau^d\tau^n(1-e^{-\Delta t^d/\tau^d}) - \tau^n\Delta t^d e^{-\Delta t^d/\tau^d} \\
&= \tau^n\tau^d - \tau^n e^{-\Delta t^d/\tau^d}(\tau^d + \Delta t^d) \\
&= \tau^n[\tau^d - e^{-\Delta t^d/\tau^d}(\tau^d + \Delta t^d) \\
&= \tau^n[\tau^d - (\tau^d + \Delta t^d)e^{-\Delta t^d/\tau^d}] \\
&= \tau^n[\tau^d(1-e^{-\Delta t^d/\tau^d}) - \Delta t^d e^{-\Delta t^d/\tau^d}] \qquad \text{Q.E.D}
\end{aligned}
$$

The exact solution for the average useful daytime (nighttime) gain Q_d (Q_n) for a direct system employing night time insulation with attached solarium is completed in (A12).

A16 Empirical fit of outlet to inlet area on perCent increase of air flow due to wind and thermal forces $\dot{V}$ wind, $\dot{V}$ stack

The desired fit on the curve for $\partial \dot{V}_{wind}$ ($\partial \dot{V}_{stack}$) vs A_{out}/A_i will be assumed of the form

$$\left(\frac{A_{out}}{A_i} - 1 \right) = A_1 \, \partial \dot{V}^{A_2} \qquad -(A\,16\cdot1)$$

where A_{out} and A_i represent the ventilation outlet area and inlet respectively in m^2. $\partial \dot{V}$ is the percentage increase in air volume flow rate (m^3/s) (wind and stack forces). A_1 and A_2 are constants of the fit. See curve (17). Equation A16·1 reduces to

$$\mathrm{Log} \left(\frac{A_{out}}{A_i} - 1 \right) = \mathrm{Log}\, A_1 + A_2 \, \mathrm{Log}\, \partial \dot{V} \qquad -(A\,16\cdot2)$$

This is the linear form in the log $\left(\frac{A_{out}}{A_i} - 1\right)$ and log $\partial \dot{V}$ variates with log A_1 being the intercept on the log $\left(\frac{A_{out}}{A_i} - 1\right)$ axis. A_2 is the slope of the curve. Equation A16.2 is a linear form, and linear regression yields the coefficients

$$\mathrm{Log}\, A_1 = \overline{\mathrm{Log} \left(\frac{A_{out}}{A_i} - 1 \right)} - A_2 \, \overline{\mathrm{Log}\, \partial \dot{V}} \qquad -(A\,16\cdot3)$$

and

$$A_2 = \frac{\sum \mathrm{Log}\, \partial \dot{V} \cdot \mathrm{Log} \left(\frac{A_{out}}{A_i} - 1 \right) - \frac{1}{n} \left(\sum \mathrm{Log}\, \partial \dot{V} \right) \left(\sum \mathrm{Log} \left(\frac{A_{out}}{A_i} - 1 \right) \right)}{\sum \left(\mathrm{Log}\, \partial \dot{V} \right)^2 - \left(\sum \mathrm{Log}\, \partial \dot{V} \right)^2 / n}$$

where n is the data number. $-A(16\cdot4)$

using equations A16.3 and A16.4 and data set from the curve (17) log A_1 and A_2 , are easily evaluated as

$$A_2 = 2.7 \qquad -(A16.5)$$

$$\text{Log } A_1 = -3.74 \qquad -(A16.6)$$

From equation 16.2, 16.5 and 16.6

$$\text{Log}\left(\frac{A_{out}}{A_i} - 1\right) = -3.74 + 2.7 \text{ Log } \partial\dot{V} \qquad -(A16.7)$$

$$\text{or } \log\left(\frac{A_{out}}{A_i} - 1\right) = \text{Log}\left(10^{-3.74} \cdot \partial\dot{V}^{2.7}\right) \qquad -(A16.8)$$

$$\text{and } \left(\frac{A_{out}}{A_i} - 1\right) = 1.83 \times 10^{-4}\, \partial\dot{V}^{2.7} \qquad -(A16.9)$$

Rearranging equation A16.9 yields

$$\frac{10^4}{1.83}\left(\frac{A_{out}}{A_i} - 1\right) = \partial\dot{V}^{2.7} \qquad -(A16.10)$$

$$\text{and } \partial\dot{V} = \left(\frac{10^4}{1.83}\right)^{0.37}\left(\frac{A_{out}}{A_i} - 1\right)^{0.37} \qquad -(A16.11)$$

$\partial\dot{V}$ in equation A16.11 is a percentage increase, therefore

$$\partial\dot{V} = \frac{24.15}{100}\left(\frac{A_{out}}{A_i} - 1\right)^{0.37} \qquad -(A16.12)$$

Fractional increase.

From which

$$\partial\dot{V} = 0.2415\left(\frac{A_{out}}{A_i} - 1\right)^{0.37} \qquad -(A16.13)$$

Fractional increase

Equation A16.13 presents us with the relation between the outlet, inlet area (A_{out}/A_i) and the fraction increase in ventilation volume flow rate (m^3/s)(due to unequivalent inlet, outlet areas). The areas measured in m^2 .

The Order of Fit

In evalauating the fit, one determines the correlation coefficient of the $\partial\dot{V}$ variate on the A_{out}/A_i variate. The correlation coefficient being

$$r_c = \frac{\sum_{j=1}^{n}\left[\frac{A_{out}}{A_i}\Big|_j - \overline{\frac{A_{out}}{A_i}}\right]\left[\partial\dot{V}\big|_j - \overline{\partial\dot{V}}\right]}{\left[\sum_{j=1}^{n}\left(\frac{A_{out}}{A_i}\Big|_i - \overline{\frac{A_{out}}{A_i}}\right)^2\right]^{0.5}\left[\sum_{j=1}^{n}\left(\partial\dot{V}\big|_j - \overline{\partial\dot{V}}\right)^2\right]^{0.5}} \quad -(A16.4)$$

Using the predicted data sets one obtains

$$r_c = 0.94 \quad -(A16.5)$$

Such a value of the coefficient of correlation indicates that the power form of equation A16.3 predicts the actual data sets with as good enough accuracy as any other. An ideal fit is characterised by a coefficient ($r_c = 1.0$).

Table A16.1 has been constructed for purposes of comparison. It is seen that the predicted values by equation A16.13 compare sufficiently well with the actual data records. It is of particular interest to mention that the closest accuracy in prediction of $\partial\dot{V}$ occurs when the ratio between inlet and outlet area A_{out}/A_i or vice versa is numerically four. This presents us with a possible design choice namely the area of the stack in the roof space (exit) is a fourth the inlet area (equivalent to area of open doors or windows in conservatory) in m^2 .
Thus one selects,

$$A_i = 4 \times A_{out}$$

when using equation A16.13, and is reasonably safest. Figure 2.7 shows a plot of the predicted and monitored records.

A_{out}/A_{in}	(*) (%) $\Delta\dot{V}_{stack}$ & $\Delta\dot{V}_{wind}$	(+) (%) $\Delta\dot{V}_{stack}$ & $\Delta\dot{V}_{wind}$
1.5	17.5	18.7
2.0	27.5	24.2
3	34	31.2
4	36	36.3
6	38	43.8

(*) Ref.(17) curve,

(+) Curve fit; eqn. 2.51; (r = 0.94), see A16.

Table A16.1 Effect of outlet to inlet area A_{out}/A_{in} on air flow due to wind and thermal forces, ($\dot{V}_{stack}$) & ($\dot{V}_{wind}$) (m^3/s).

A17 Fitted Curve for Ventilation air flow rate(m^3/s) due to combined wind and stack forces as function of stack to combined flow rates (Fig 2.9)

The curve to be fitted is presented in the ASHRAE handbook of fundamentals (17). The empirical form will be written as

$$\left[100\left[\frac{\dot{V}_{stack}}{\dot{V}_{stack}+\dot{V}_{wind}} - 0.1\right]\right]^{a}\left[F_m - 1\right] = b \qquad \text{– (A 17.1)}$$

where $\dot{V}_{stack}$ is the ventilation air flow rate due to thermal forces (m^3/s); $\dot{V}_{wind}$ is the ventilation air flow rate due to wind forces (m^3/s). F_m is the actual flow multiple on flow due to temperature difference. a and b are constants of the equation. Writing the second term of the L.H.S. of eqn. A17.1 as D,

$$\left[100\left[D - 0.1\right]\right]^{a}\left[F_m - 1\right] = b \qquad \text{– (A 17.2)}$$

In eqn. A17.2, D is the x variate and F_m the y-variate. The equation A17.2 has two assymtodes on the lines
D = 0.1 and F_m = 1, respectively.

The logarithmic form of equation A17.2 is

$$\mathrm{Log}\left[100\left[D - 0.1\right]\right]^{-a} \cdot b = \mathrm{Log}\left[F_m - 1\right] \qquad \text{–(A 17.3)}$$

or by logarithmic properties

$$\mathrm{Log}\left[F_m - 1\right] = \mathrm{Log}\, b - a\,\mathrm{Log}\left[100\left[D - 0.1\right]\right] \qquad \text{–(A 17.4)}$$

Equation A17.4 above is a linear form. The y variate is log (Fm-1), while the x-variate is log (100(D − 0.1)). Log b and a are constants of the equation.

A linear regression of equation A17.4 yields the constants

$$\text{Log}\, b = \overline{\text{Log}\,[F_m - 1]} - a\,\text{Log}\,[100\,[D - 0.1]] \quad -(A17.5)$$

and

$$a = \frac{\Sigma\,(\text{Log}\,[F_m - 1]\,\text{Log}\,[100\,[D - 0.1]\,]) - (\Sigma\,\text{Log}\,[F_m - 1])(\Sigma\,\text{Log}\,[100(D - .1)])}{\Sigma\,(\text{Log}\,100\,[D - 0.1])^2 - (\Sigma\,\text{Log}\,100\,[D - 0.1])^2/n}$$

$$-(A17.6)$$

where n is the number of data sets.

Using equation A17.5 and A17.6 together with the data set (Ref.17). log b and a are evaluated as

$$\log b = 2.526 \quad - (A17.7)$$

$$a = 1.9 \quad - (A17.8)$$

From equations A17.4, A17.7 and A17.8

$$\text{Log}\,[F_m - 1] = 2.526 - 1.9\,\text{Log}\,[100\,[D - 0.1]\,]$$

$$- (A17.9)$$

or $\log\,[F_m - 1] = \text{Log}\,10^{2.526}\,[100\,[D - 0.1]\,]^{-1.9}$

$$-(A17.10)$$

and

$$[F_m - 1] = 335.5\,[100\,[D - 0.1]\,]^{-1.9}$$

$$-(A17.11)$$

Rearrange eqn. A17.11 and

$$F_m = \frac{355.5}{\left[100\left[\dfrac{\dot{V}_{stack}}{\dot{V}_{stack} + \dot{V}_{wind}} - 0.1\right]\right]^{1.9}} + 1$$

$$- (A17.12)$$

The coefficient of correlation

$$r_c = -0.97 \quad - (A17.13)$$

Again such value for the fit order T_a indicates equation A17.12 can be used with a comfortable level of confidence.

F_m as given by equation A17.12 is the multiplication factor on the ventilation air flow rate due to thermal forces (m^3/s). This product for a given ratio of stack to combined ventilation flow rates represents the combination flow rate when stack and wind forces are considered. In (m^3/s)

$$\dot{V}_{vent, wind, stack} = \left[\frac{355.5}{\left[100\left[\frac{\dot{V}_{stack}}{\dot{V}_{stack} + \dot{V}_{wind}} - 0.1\right]\right]^{1.9}} + 1 \right] \dot{V}_{stack} \qquad -(A\,17.14)$$

where $\dot{V}$ vent, wind, stack is obtained in (m^3/s) and $\dot{V}$ stack ($\dot{V}$ wind) represent the volumetric flow rates by thermal and wind effects in (m^3/s).

Another inspection of equation (A17.12) reveals that F_m is a decaying function of stack to combined flow rates. This function decays to unity at sufficiently large values of stack to combined flow rates. Thus, for predominant stack flow conditions equation A17.14 reduces to

$$\dot{V}_{vent, wind, stack} = \dot{V}_{stack} \qquad -(A\,17.5)$$

as should be expected.

Table A17.1 below has been constructed for purposes of comparing the predictions by equation A17.12 and the monitored values (ref.17) for various stack to combined flow rates.

$\frac{\dot{V}_{stack}}{\dot{V}_{stack} + \dot{V}_{wind}}$ %	F_m *	F_m +
20	4	5.1
30	2.5	2.095
40	1.75	1.505
50	1.25	1.291
60	1.2	1.189
70	1.1	1.134

(*) Ref [17]

(+) Eq(A17.12)

$r_c = -0.97$

Table A17.1. Predicted and monitored values of the multiplication factor F_m.

Again, a choice of $\frac{\dot{V}_{stack}}{\dot{V}_{stack} + \dot{V}_{wind}}$ = 50-70% best suits the empirical form A17.12. This suits situations normally encountered in practice, as the stack effects will more commonly dominate the wind forces.

A18 Solution of the Energy Balance Equation of Building Envelop

The following equation is to be solved (eqn.2.6)

$$\dot{m}C_p \frac{dT_f}{dt} - \left[(BF_c Q_T + (1-B)FQ_T) - U_{eq}(T_f - T_a)\right] = 0 \quad (A18{\cdot}1)$$

OR

$$\frac{dT_f}{dt} + \frac{U_{eq}}{\dot{m}C_p} T_f - \frac{(BF_c Q_T + (1-B)FQ_T)}{\dot{m}C_p} - \frac{U_{eq}T_a}{\dot{m}C_p} = 0 \quad (A18{\cdot}2)$$

of form

$$\frac{dy}{dx} + C_1 y - C_2 = 0 \quad (A18{\cdot}3)$$

$$\frac{dy}{dx} + C_1 y = C_2 \quad (A18{\cdot}4)$$

$$(\times e^{C_1 x}) \quad \therefore \frac{dy}{dx} e^{C_1 x} + y \cdot C_1 e^{C_1 x} = C_2 e^{C_1 x} \quad (A18{\cdot}5)$$

OR

$$\frac{d}{dx}(y \cdot e^{C_1 x}) = C_2 e^{C_1 x} \quad (A18{\cdot}6)$$

Integrating both sides w.r.t. x

$$y e^{C_1 x} = \frac{C_2 e^{C_1 x}}{C_1} + c \quad (A18{\cdot}7)$$

$$(\div e^{C_1 x}) \quad y = \frac{C_2}{C_1} + c e^{-C_1 x} \quad (A18{\cdot}8)$$

$$\therefore T_f = \frac{BF_c Q_T + (1-B)FQ_T + U_{eq}T_a}{(\dot{m}C_p)(U_{eq}/\dot{m}C_p)} + Ce^{-U_{eq}t/\dot{m}C_p} \quad (A18{\cdot}9)$$

or $$T_f - T_a = (BF_c Q_T + (1-B)FQ_T)/U_{eq} + Ce^{-U_{eq}t/\dot{m}C_p} \quad (A18{\cdot}10)$$

We apply the condition that at inlet, when $t = 0$, the fluid temperature is the conservatory temperature T_{co}.